A Method to Assess the Progressive Collapse Vulnerability of Frame Structures

Dissertation

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by
Francesco Barni
Born 10.10.1972
from Prato, Italy

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Professorial advisors	Prof. D. Dinkler
	Prof. C. Borri
	Prof. G. Bartoli

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*) Either the German or the Italian form of the title may be used.

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Used notation

Acronyms

ACI	American Concrete Institute
AEM	Applied Elements Method
AISC	American Institute of Steel Construction
ALP	Alternate Load Path (mitigation strategy)
ANSI	American National Standards Institute
APDL	ANSYS Parametric Design Language
ASCE	American Society of Civil Engineers
CSF	Central Safety Factor
DoD	Department of Defense
EC	Event Control (mitigation strategy)
FE	Finite Element
FEM	Finite Elements Method
FEMA	Federal Emergency Management Agency
FORM	First Order Reliability Method
GSA	General Services Administration
IDM	Indirect Design Method (mitigation strategy)
IMF	Intermediate Moment Frame
JCSS	Joint Committee on Structural Safety
LRFD	Load and Resistance Factor Design
MPC	MultiPoint Constraint (finite element type)
NEHRP	National Earthquake Hazards Reduction Program
NIST	National Institute of Standards and Technology
SLR	Specific Load Resistance (mitigation strategy)
SDOF	Single Degree Of Freedom oscillator
SORM	Second Order Reliability Method
WUF-B	Welded Unreinforced Flange-Bolted Web connection type
WTC	World Trade Center

Measurement units

kPa	kiloPascal	1kPa=1000 N/m^2
MPa	MegaPascal	1MPa=10^6 N/m^2
m	meter	
mm	millimeter	1mm=0.001 m
N	Newton	
psi	pounds per square inch	1psi≈6894.757 N/m^2
yr	year	
ksi	kilo-pound per square inch	1ksi=1000 psi
kg	kilogram	
GN	GigaNewton	1GN=10^9 N

Parameters

Note: this list does not include parameters reported in chapters 1, 2 and 3, where several works and regulations are quoted and the same nomenclature denotes different parameters.

A	generic definite positive matrix
AR	acceptability range
C	Collapse; event that the initial Local Damage spreads

C_{ik} event that Collapse happens due to the k-th scenario of the i-th Local Damage level
D dead load
E relative error or relative difference
FD Final Damage
FD_{ik} Final Damage level due to the k-th scenario of the i-th Local Damage level
G($\mathbf{y}$) transformed of the performance function g($\mathbf{x}$) into the standard Gaussian space
H Hazard
HI Hazard Intensity
HI_{crit} critical value of the Hazard Intensity
IP parameter of interest
IP_A actual value of a parameter of interest
IP_C calculated value of a parameter of interest
L live load
LD Local Damage;
LD_i event that a Local Damage scenario of level i happens
LD_{ik} event that the k-th scenario of the i-th Local Damage level happens
P( ) probability mass function, i.e. a function that gives the probability that a discrete random variable is exactly equal to some value
$P(\ )_{acc}$ acceptable value of a probability
$P(\ )_{max}$ estimated maximum value of a probability
$P(\ )_{min}$ estimated minimum value of a probability
P_f probability of failure
R resistance of a structure, i.e. the applied vertical load that prompts Collapse
S total applied vertical load on a structure
V_i coefficient of variation of the i-th random variable
$\mathbf{d_i}$ i-th generic vector
f($\mathbf{x}$) joint probability density of $\mathbf{x}$
f considered Final Damage level
f_y yielding stress of steel
g($\mathbf{x}$) performance function, i.e. a function that assumes positive values if a given performance is fulfilled and negative values if it is not
i generic counter
j generic counter
k generic counter
m total number of random variables
n total number of structural elements in a structure
p number of calculated damage levels
p(x) probability density function of random variable x
r radius (polar coordinate)
$r^{(i)}$ value of radius (polar coordinate) at the i-th step of an algorithm
s step length of a gradient method
t total number Local Damage combinations for a given Local Damage level
$\mathbf{u}$ generic vector
$\mathbf{x}$ generic vector of random variables
$\mathbf{x}^{(i)}$ vector of random variables at the i-th step of an algorithm
x_i i-th entry of vector $\mathbf{x}$
x generic of random variable
$\mathbf{y}$ transformed of vector $\mathbf{x}$ into the standard Gaussian space
$\mathbf{y}^{(i)}$ transformed of vector $\mathbf{x}^{(i)}$ into the standard Gaussian space
y_i i-th entry of vector $\mathbf{y}$

$\mathbf{y^*}$ point of the limit state surface closest to the origin of the standardized space
z displacement of a Single Degree Of Freedom oscillator

Φ cumulative distribution function of the standard Gaussian distribution
$\boldsymbol{\alpha}$ vector of sensitivity factors
α_i i-th entry of $\boldsymbol{\alpha}$
β reliability index
χ_n^2 chi-square distribution with n degrees of freedom
φ probability density function the standard Gaussian distribution
$\boldsymbol{\varphi}$ vector of angles (polar coordinates)
φ_i i-th entry of $\boldsymbol{\varphi}$
$\boldsymbol{\varphi}_i$ i-th vector of angles (polar coordinates)
μ_i mean value of the i-th random variable
σ_i standard deviation of the i-th random variable
ξ uncertainty of a structural model

Introduction

> "*When you can measure what you are speaking about, and express it in numbers, you know something about it; but when you cannot measure it, when you cannot express it in numbers, your knowledge is of a meager and unsatisfactory kind: it may be the beginning of knowledge, but you have scarcely, in your thoughts, advanced to the stage of science.*" –William Thomson, 1st Lord Kelvin [48].

Every structure can suffer a localized Damage because of some unforeseen event. This localized Damage can either remain confined or spread up to a final wider extent. The latter case is commonly known as Progressive Collapse.

Of course, Progressive Collapse is not desirable. Thus, for over forty years building regulations have been asking to avoid or limit this phenomenon, and several mitigation techniques have been identified. Yet research on Progressive Collapse mitigation is still in a primordial state.

When designing for other types of Hazard (such as for example earthquake or wind), us designers usually follow some established methodologies: we get information about the characteristics of the Hazard; we choose the characteristics of the structure to be made, such as material types and element dimensions; we make a model of the structure, and with this model we calculate some parameters. By comparing these parameters with parameters derived from the Hazard, we can assess if the structure is to be deemed verified. In modern design methods, “verified” usually means that the probability of occurrence of some unwanted events (such for example a building collapsing under an earthquake) is lower than an acceptability value.

With Progressive Collapse a similar procedure still does not exist. We still don't have reliable and practical methods to quantify the propensity to Progressive Collapse of a given structure, as well as to quantify an acceptable level of this propensity. In other words, we have the tools to mitigate, but we still don't have a reliable methodology to decide *if* and *how much* we need to apply them. In the words of Lord Kelvin, our knowledge is “*of a meager and unsatisfactory kind*”.

The present work is an attempt to solve this problem by incorporating Progressive Collapse in a probabilistic Risk Framework.

The motivations and the target of the present work are presented and discussed in section 5.1.
The first part of the work (chapters 1, 2, 3 and 4) provides information useful to understand what the problem consists of, as well as to better understand the approach proposed to solve it.
The second part (chapters 5, 6, 7 and 8) presents the proposed approach, test applications of it and ideas for further developments.

More in detail:
Chapter 1 introduces the concept of Progressive Collapse, lists the characteristics that are desirable in a structure in order to mitigate this phenomenon, and lists causes of Collapse, subdivided in categories. Furthermore, it describes and analyzes some of the most important case studies.

Chapter 2 lists and explains the strategies that have been devised for Progressive Collapse mitigation and lists some of the most significant regulations about this subject, explaining how they evolved.

Chapter 3 lists and analyzes several ideas that have been proposed for parameters to quantify the propensity to Progressive Collapse of structures and elaborates on the reasons why no building code has adopted any of these methodologies yet.

Chapter 4 introduces the concept of Risk. It describes the probabilistic Risk management framework developed by the University of Braunschweig, whose concepts are used in this work, and points out some aspects of the adopted nomenclature.

Chapter 5 proposes two methodologies to quantify Progressive Collapse propensity of frame structures, which basically consist in incorporating this phenomenon in a probabilistic Risk framework. First, the motivations to the development of these methodologies and the targets they aim to are explained. Then, the basic ideas and the details of how the methodologies can actually be implemented are described.

Chapter 6 presents some test applications of the first proposed methodology. It includes the description of the used structural models and of how the single elements of the framework are implemented. Examples of analyses are presented.
It is pointed out that the main target of the performed analyses is not the testing of the modeled structures, but the testing of the algorithms used for the analyses, to study the the feasibility of the proposed approaches, to find out their problems, and to debug the implemented algorithms.

Chapter 7 summarizes the entire work and presents its conclusions.

Chapter 8 lists several aspects that need to be further considered to improve the proposed methodologies. Other ideas that might be developed are also described.

The conclusions of the work are presented in section 7.2.
No particular evidence was found to prove that the proposed target is impossible to reach through the adopted approach. It is concluded that the present work represents the first steps towards the achievement of the proposed target, and that the developed methodologies are to be considered as prototypes that need further improvement to actually be usable.

Chapter 1 - What is Progressive Collapse

This chapter reports information useful to understand the problem this work is about, as well as to better understand other parts of the work.

Section 1.1 gives a definition of Progressive Collapse and elaborates on it.
Section 1.2 lists the characteristics that are desirable in a structure in order to reduce the chances of occurrence of Progressive Collapse, or to mitigate its intensity.
Section 1.3 lists causes of Collapses, subdivided in categories.
Section 1.4 describes and analyzes some of the most important case studies of Progressive Collapse, as well as a significant case in which an initial Damage did not evolve in a Collapse; references to characteristics and causes listed in sections 1.2 and 1.3 are given.
Section 1.5 summarizes the chapter and elaborates on aspects that will be useful in the rest of the work.

1.1 Definition of Progressive Collapse

Several slightly different definitions of the expression "Progressive Collapse" have been formulated. One of the most used is given by the US Standard ASCE 7 [6]: "*Progressive Collapse is defined as the spread of an initial local failure from element to element, eventually resulting in the collapse of an entire structure or a disproportionately large part of it*".

The common feature of all the definitions is that an initial Damage that is limited in space progresses up to a bigger final extent. The underlying idea is that the initial Damage is provoked by some traumatic event (such e.g. an explosion, a collision or a localized deterioration) while the extra Damage is not directly due to the traumatic event, but due to the inability of the wounded structure to bear its loads.

Thus, in a Progressive Collapse two stages can be identified.
In the first one, the structure undergoes a traumatic event and suffers some local Damage. The local Damage is the direct effect of the traumatic event, so in this phase the effect is considered proportionate to the cause.
In the second stage the Damage progresses up to a final wider extent. This second type of Damage is an indirect effect of the traumatic event.
The total Losses due to the traumatic event are the sum of those due to the direct and indirect effects. In some cases the indirect effects are much bigger than the direct effects; thus, a relatively small traumatic event can potentially lead to relatively big Losses. Since the possibility of occurrence of a local Damage cannot be completely avoided, it is desirable to limit the indirect effects.

One flaw that has been highlighted (for example in section 2.1 of [35]) is that most definitions of Progressive Collapse include the terms "disproportionately" or "disproportionate", which makes them ambiguous. While these terms express an important aspect of the idea of Progressive Collapse, there is not general consensus on when a Damage is to be considered "disproportionate", so the same Collapse could be subjectively considered Progressive or not. Furthermore, this ambiguity does not make clear what are the targets to pursue in Progressive Collapse mitigation.

The ambiguity has led to some debate. For example, according to some Authors (section 2.4 of [35]) the Murrah Building Collapse (which is thoroughly described in section 1.4.3 of the present script) does not comply with the definition given by ASCE 7, even though in literature it is generally presented as one of the most significant examples of Progressive Collapse.

In the present work the question of what is “disproportionate” is not addressed. Instead, the target is to estimate if the consequences of the events are acceptable or not, independently from the ratio between indirect and direct consequences. This is done by incorporating Progressive Collapse in a probabilistic Risk framework.
Referring again to the Murrah Building Collapse, in the Writer's opinion the important thing to consider is that about 80% of the 168 victims, which were not directly due to the explosion, could have been be avoided [40], regardless of the chosen nomenclature.

Figure 1.1: Three examples of structures that suffered localized traumatic events, with different outcomes. The building on the left suffered direct Damage from an explosion; the building in the center partially collapsed because of a car impact; the structure on the right completely collapsed during construction due to inadequate bracing (Source: [4]).

1.2 Opposing Progressive Collapse

In order to reduce the chances of occurrence of Progressive Collapse, or to mitigate its intensity, some characteristics are desirable in a structure.

Resistance is the ability of a body or a material of not breaking under loading.

Stability is the property of a system of being able to withstand an arbitrary limited perturbation with a limited variation of its state. In the present context, we are referring to the static equilibrium stability of a given structure.

Ductility is the ability of a body or a solid material to endure permanent deformations without breaking. The permanent deformations usually compromise the functionality of a structure, but they allow energy dissipation; thus ductility is generally considered a desirable quality, especially in seismic design. Historically, ductility began to be considered only after the adoption of structural steel and reinforced concrete in the nineteenth century, because the natural materials that have been used for thousands of years (mainly wood and stone) don't have this quality.

In the present context, *Redundancy* is the quality of a structure to be able to sustain its loads in more than one way, i.e. to convey its loads to the ground through multiple load paths. According to M. Levy and M. Salvadori [22] "*redundancy must be considered a necessary quality in every big structure or in every structure whose collapse can cause major damage or loss of human lives*" and "*all structural collapses can be considered as due to the lack of redundancy*". Before the Ronan Point disaster (which is described in section 1.4.1 of the present script) few building codes included prescriptions for redundancy, even though its utility was known (like in the case of Eads Bridge, described in section 1.4.5).

There is generally consensus about the fact that structural safety also depends on *accuracy* of design and construction of structural details.

Furthermore, *compartmentalization* is the technique of subdividing a structure in parts, or "compartments", and apply means to avoid the transmission of the Damage from one compartment to the other. This is generally achieved by disconnecting each compartment from the others or, more rarely, by strategically making some parts of the structure very resistant. In the first case, if a compartment gets damaged, in the worst case scenario the entire compartment collapses, but the other ones are not affected because they are not connected to it. In the second case, if some Damage ensues, the Damage should reach the reinforced parts and stop. Figure 1.2 depicts a case in which compartmentalization by disconnection was used in the design of a long bridge.

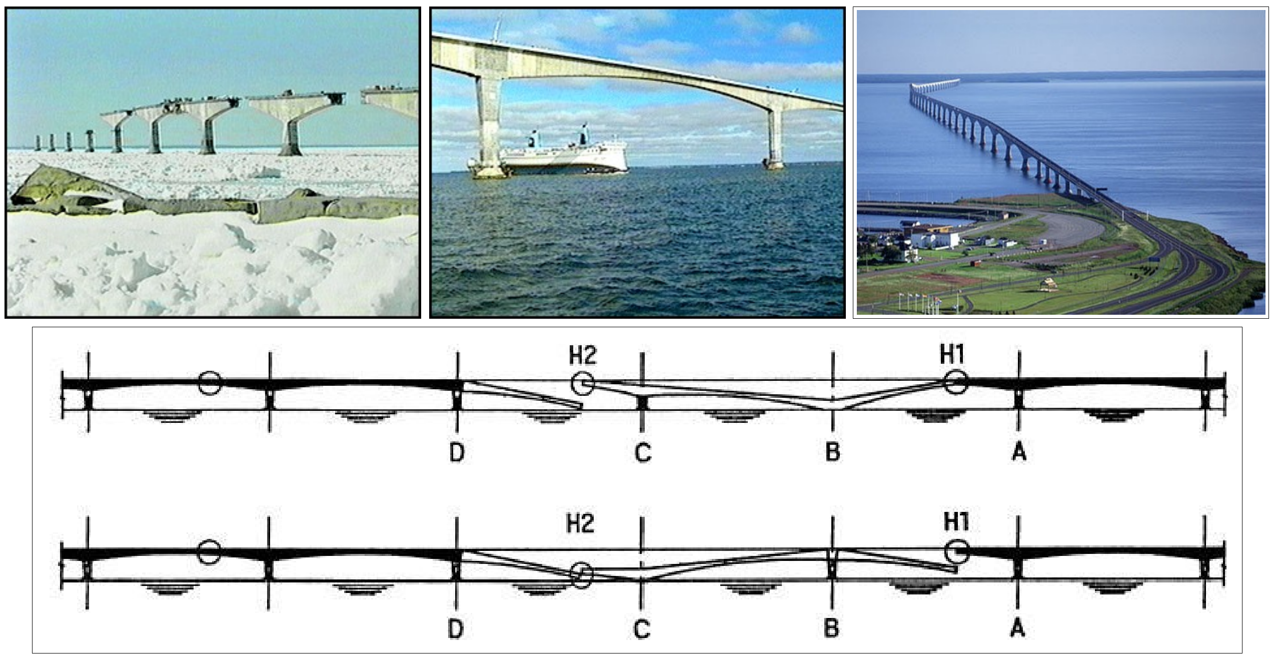

Figure 1.2: The Confederation Bridge in Prince Edward Island, Canada, is a post-tensioned prestressed concrete bridge composed of 43 spans, each 250m long. It was inaugurated in 1997. It was designed with structural "fuses", to allow the maximum Collapse of two and a half spans in case of one structural element failure. The idea to apply complementation arose by observing that in 1992 a similar bridge in Seoul, Korea suffered a Progressive Collapse in which the prestressing cables ripped the concrete decks for a total of eleven spans. (Sources: www.confederationbridge.com; [46]).

It must be observed that redundancy and compartmentalization by disconnection are considered antithetical, in that the first requires a high level of connection and the second requires strategically designed disconnections.
In general, compartmentalization is more suitable for structures with a prevalent horizontal extension, in which providing multiple load paths (i.e., providing redundancy) is difficult or not

possible at all. Compared to redundancy, compartmentalization might require the sacrifice of an entire compartment to save the rest of the structure (like in the example of figure 1.2), while in the best case scenario a redundant structure is able to suffer a localized Damage without any spreading of it (like in the case of figure 1.1, left). However, in general a highly connected structure is not automatically a redundant structure. If the parts that surround a damaged area are not able to bear the increased loads, they can be "dragged down" by it. Thus, in some cases a high level of connection can be a disadvantage.

1.3 What prompts a Collapse

When a Collapse happens, it is usually due to a concurrence of causes. In many cases it is possible to recognize a main cause and/or prompting event (usually this is done with investigations in legal actions). These causes can be subdivided in categories, as listed next.

During the design phase:

- models not adequate to the physical system actually built (like in the example of figure 1.3);
- wrong design. This condition happens when the used models are adequate, but are applied in the wrong way or there are big calculation errors;
- not consideration of some loads.

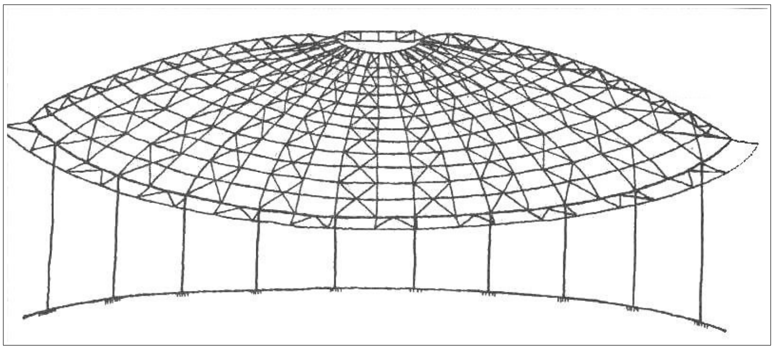

Figure 1.3: The C.W. Post College dome was located in Long Island, New York, USA. It was 51m in diameter. Its structure consisted in steel parallels, meridians and x-shaped braces arranged in alternate meridian sectors. It collapsed in the early morning of the 21st of January, 1978; the prompting event was an asymmetrical accumulation of ice and snow, which caused the progressive buckling (a form of instability) of the steel elements. The subsequent investigations found out that the structure had been designed with a too simplified theory, which had the hypotheses of isotropic materials and symmetrical vertical loads; an asymmetrical vertical load, of about one quarter of the considered design load, would have been enough to prompt a Collapse. (Source: [22]).

During the construction phase

- bad workmanship;
- low quality materials, compared to those specified in the design;
- differences between the design and what is actually built.

During use:

- loads unforeseen in the design phase. In this category are generally included exceptional loads like impacts and explosions, but also cases of misuse of a structure such as buildings improperly used as storehouses, in which excessive loads are imposed;
- materials deterioration (corrosion, fire);
- bad maintenance or modifications to the structure.

Structural Collapse can also happen during construction or demolition phases, thus it is often necessary to "design" and "verify" these phases, too.

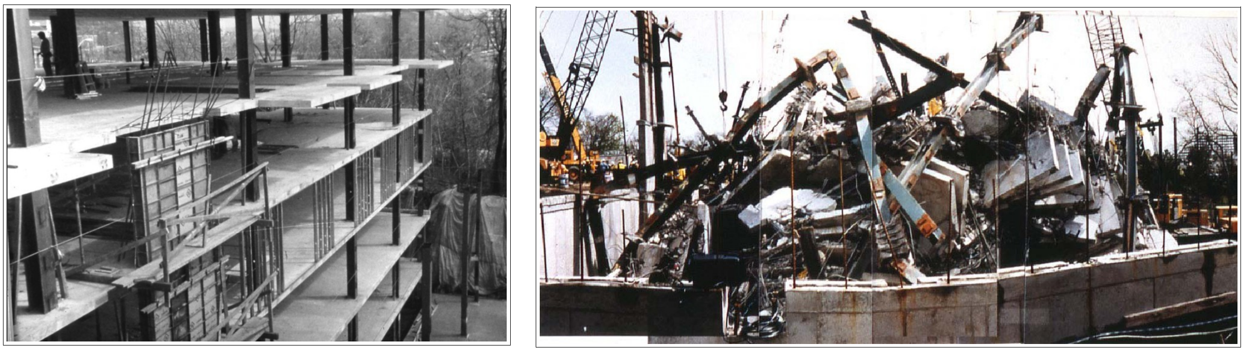

Figure 1.4: The Collapse of the Ambiance Plaza in Bridgeport, Connecticut, USA, is probably the most famous case of Progressive Collapse during construction. The structure was supposed to become a sixteen-story apartment building. Its construction technique, called "lift-slab", consisted in casting the reinforced concrete floor slabs on the ground and then lift and anchor them to the steel columns.
On the 23rd of April, 1987, several slabs collapsed progressively one on the other, causing the death of 28 workers. The most likely prompting event was the failure of a lifting device. (Source: NIST).

1.4 Case studies

This section reports five important cases of Progressive Collapse, as well as a meaningful case in which an initial Damage did not result in Collapse. Each Collapse case is described and analyzed, and their consequences (in terms of research and building codes implementation) are reported. At the end of the chapter, the most important aspects of the cases are summarized and discussed.

1.4.1 The Ronan Point Tower

On the 16th of May 1968, at 5:45 AM, an explosion occurred at the eighteenth floor of the building denominated Ronan Point Tower, located in Canning Town, England. This prompted a chain-like Collapse, which damaged all 22 stories of the building, causing the death of 4 people. This episode made history; following it the expression "Progressive Collapse" was invented, and research begun aimed at preventing it or at mitigating its effects.

Figure 1.5: The Ronan Point Tower and its Collapse mechanism. (Sources: NIST, www.imacleod.com).

Description
The tower was built between 1966 and 1968 and was part of a group of nine twin buildings. The floor plan was rectangular and repeated identically on every floor. The towers were built using a new (at the time) pre-casting system, which had the advantage of being fast and economical. The system was composed of reinforced concrete panels, each extended as an entire room, for walls and floor slabs. It required that the connections between the panels were to be filled with mortar, but without additional metal ties.

Analysis
The explosion was prompted by a gas leak on the eighteenth floor. The investigations concluded that one external vertical panel, which had both structural and separation functions, was expelled by the overpressure caused by the explosion. The panels located above the expelled one, lacking support for their weight, started to fall. When they hit the elements of the seventeenth floor, the acquired kinetic energy was enough to break them. The same thing happened on all the underlying floors.

One surprising fact is that, according to the investigations, the wall panel from which the Collapse started did not break; instead, it was expelled from its housing. The woman who caused the explosion while trying to turn on her stove was thrown towards the inside of the building and survived. Since she did not suffer permanent ear damage, it was concluded that the pressure of the explosion had to be less than $0.07 N/mm^2$, which is roughly equivalent to the pressure 3m underwater. Experimental tests showed that $0.02 N/mm^2$ would have been enough to expel that particular panel; if the explosion had happened at a lower floor, the friction with the surrounding elements could have retained the panel.

The structure was an ideal candidate for a Progressive Collapse: its connections were fragile; the capability to absorb energy by deformation (ductility) was lacking; it had no capability to redistribute its loads after a local Damage (redundancy); its size was considerable and its conformation was primarily vertical.

Consequences
The inquiry report of the Ronan Point disaster highlighted the need to improve building regulations in order to prevent similar events. Consequently, in November 1968 the first UK regulation that directly addressed Progressive Collapse was issued ("*Flats Constructed with Precast Concrete Panels. Appraisal and Strengthening of Existing High Blocks; Design of New Blocks"* by the UK Ministry of Housing and Local Government). Previously, the regulations of some countries had included generic prescriptions that Collapse "like a house of cards" must be avoided, but Ronan Point gave a strong impulse to the legislators. In a short period of time the building regulations of all the most advanced countries were modified to include prescriptions, more or less explicit, to contrast the phenomenon. (The evolution of building regulations is thoroughly described in section 2.2 of the present script.)

The immediate consequence of the Ronan Point Collapse was the interruption of gas distribution in all similarly designed structures. The damaged part of the building was rebuilt with reinforced connections. In 1984, after some cracks in the walls appeared, the building was evacuated; in 1986 its demolition begun. The building was disassembled piece by piece to assess the quality of construction, which resulted quite poor. In particular, the connections between the precast elements, instead of being filled with mortar, were partially empty or filled with waste material. Consequently many similar buildings were deemed insecure and demolished.

1.4.2 The Hyatt Regency Walkways

On the 17th of July 1981, at 7:05PM, two walkways located in the lobby of the Hyatt Regency Hotel in Kansas City, Missouri (USA) collapsed causing the death of 114 people and injuring 180. This was the event that provoked the highest number of victims because of a structural failure in the USA.
The investigations highlighted numerous flaws, both in the design and in the construction phases. These were especially attributed to the adopted scheduling method, which was called “fast track”. It allowed to start the construction of the building before the entire design phase was completed; thus several details could be designed concurrently to the construction process.

Description and analysis

The main building of the Hotel is 35 story high, houses the 750 rooms and is topped by a revolving panoramic restaurant. At a distance of 36.6m is located a “functional block”, i.e. a 4 story building that houses restaurants, meeting rooms and other functions. Between the two buildings is located the main lobby. To allow transit between the two buildings at the second, third and fourth floors a system of walkways was designed. The initial idea was to build the walkways on columns; lately it was decided to hang them from the ceiling, for a better visual effect. The second and fourth floor walkways were located one above the other, while the walkway at the third floor was offset and parallel to the other ones.
The total span was subdivided into four parts, each about 9m long. One extremity of the walkways was fixed to the floor system of the functional block, while the other extremity was connected to the floor system of the main building trough slotted plates, to allow expansion movements. The intermediate supports were held by couples of hangers rods connected to a roof structure.

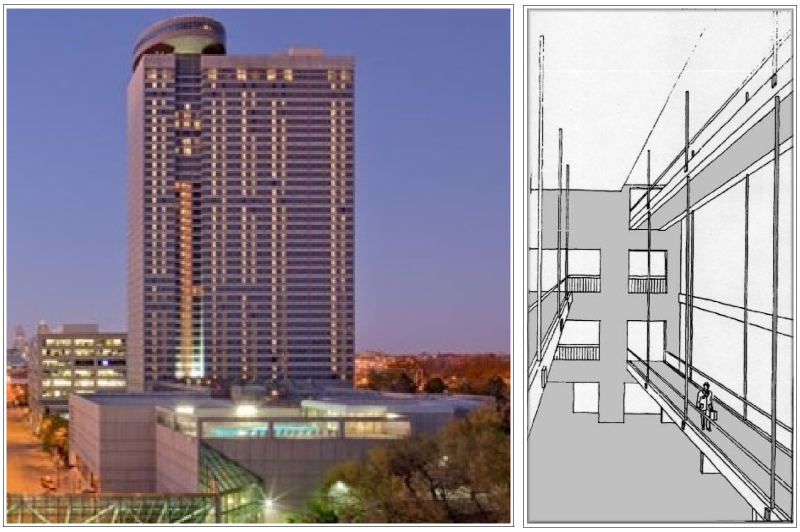

Figure 1.6: (Left) The Kansas City Hyatt Regency Hotel. The low white building in the foreground is the functional block. (Right) The original walkways in the main lobby. (Sources: crowncenter.hyatt.com, [22]).

The design of the walkways changed several times, after the construction had already started, as a consequence of the adopted “fast track” method. The design consisted in two W16x26 longitudinal beams (i.e., beams with a “double T” transverse section) topped by a metal deck and a lightweight concrete slab (figure 1.7).
Transverse beams connected the longitudinal beams and transferred the loads to the hanger rods.

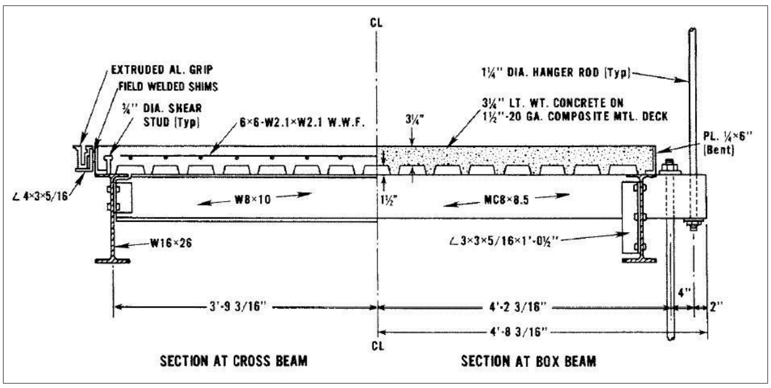

Figure 1.7: Transverse section of the walkways. On the left side is represented the first design; on the right side is represented the second design, which was built. (Source: [33]).

In the original design, the transverse beams were W8x10 (“double T” shaped; shown on the left of figure 1.7); later they were replaced with rectangular box shapes, obtained by welding flange to flange two “C-shaped” MC8x8.5 profiles (figure 1.7, right).

Figure 1.8b is the original drawing of the detail of the connection between the transverse beam and the hanger rod. According to the common practice at the time, the fact that the load on the hanger rod (22kips) is reported in the drawing indicates that the drawing is not definitive. The drawing also reports the yielding stress of the steel Fy, because the type of material had changed from the previous design. Figure 1.8c is the detail drawn by a draftsman for the documents of the contract. This time the information about the load and the material type, which implicitly suggest that the detail has not yet been designed, is not indicated.
Later, the contractor that was building the walkways asked the project manager to change the detail of the hanger rod connections upper walkway. The original idea was to use continuous steel bars, from the ceiling to the lower walkway. To facilitate construction, the contractor asked if they could be made discontinuous and offset at the fourth floor. The project manager made a quick flexural and shear verification of the box beam, while at the phone with the contractor. He considered the new solution acceptable and told the contractor to submit the request through the channels of authority; this was not actually made.

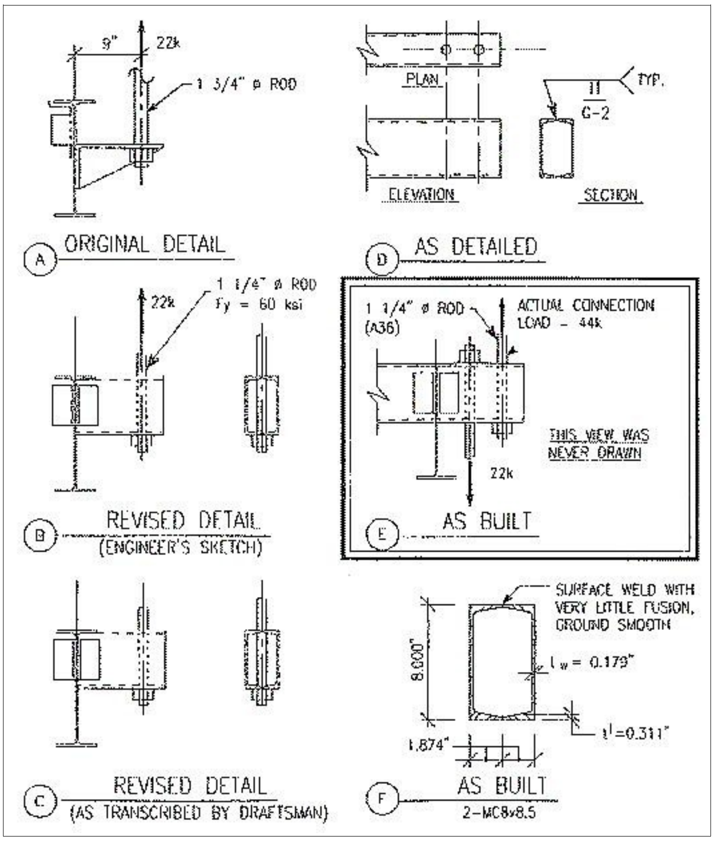

Figure 1.8: Drawings of the transverse beam/hanger rod connection, at different design stages.(Source: [28]).

A few days later the contractor pulled the job out of his engineering department to make work for another large project. The partially completed shop drawings were sent to an outside engineering company to be completed. The outside engineering company received the drawing shown in figure 1.8d. They assumed that the detail had already been designed, and only added the notation for welds to be added to keep the profiles together during shipping and erection.
The project manager, who was the only one to know the evolution of the design of the walkways, was busy and under pressure because the owner wanted the construction to be completed as fast as possible. It must be highlighted that the walkways were considered relatively unimportant, compared to the rest of the Hotel. Thus, the review of the definitive drawings was carried out by a co-worker of the project manager, who did not notice the error in the connection.
On the 14th of October 1979, when construction was completed but the hotel was not yet open to the public, one walkway partially collapsed. No one was hurt. The investigations found that the connections of the walkways with the buildings were poorly made and were not in compliance with the design. The Authorities ordered a control of the connections between walkways and buildings, but not of those with the hanger rods. Instead, it was discovered that the truss structure of the ceiling, from which the walkways were hanging, was at risk of sudden buckling, thus it had to be

modified.

Finally, in July 1980 the hotel was inaugurated.
One year later, in July 1981, one of the fourth floor hanger rod connections failed. This happened while a musical group was playing in the lobby, and the connection failure was probably prompted by the people on the walkway moving at the rhythm of the music. The connection failure basically consisted in the pulling of a rod through its connection because of the high stresses concentrated on one face of the box beam. After the first failure the loads redistributed on the other connections, which in turn failed progressively. The hanger rods holding the fourth floor walkway remained connected to the ceiling, while the second and fourth floor walkways fell together on the lobby floor.

Figure 1.9: The main lobby after the Collapse and detail of the failed connection. (Drawing source: [22]; pictures source: unknown).

The investigations that followed concluded that the connections had always worked close to their ultimate limit state, even without live loads. In the the connections with the hanger rods, the stress level in the lower flange of the beam box was so high that plastic deformations had already occurred with dead loads only. By modeling the connection with finite elements it was concluded that a simple plate between the box beam and the washer at the end of the bar would have avoided the initiating failure.

The investigations ascertained some facts:

- the connection that failed had never been designed. The project manager thought that the contractor would have designed it, as it was common practice, because he had reported in his drawing the force to withstand. Because of a copying mistake, the contractor did not receive this information;
- as a consequence, the connections between hanger rods and box beams did not meet the existing requirements;
- the walkways collapsed under very low live loads, compared to those required by the local regulations;

- modifying the connection detail from one continuous bar to two offset bars doubled the contact stress between the lower surface of the box beam at the fourth floor and the washer at the end of the bar;
- the new solution also doubled the shear load on a short span of the box beams at the fourth floor (picture 1.10b). This was deemed not determinant for the Collapse. However, if the hanger rods connecting the fourth floor walkway to the ceiling had been placed internally, the maximum shear would not have been doubled (figure 1.10c);
- material quality and workmanship did not have a determinant role in prompting the Collapse;
- in any case, even if the connections had been properly designed, the overall system lacked redundancy and ductility, thus it was not capable to contrast or limit Progressive Collapse in the case of a connection failure.

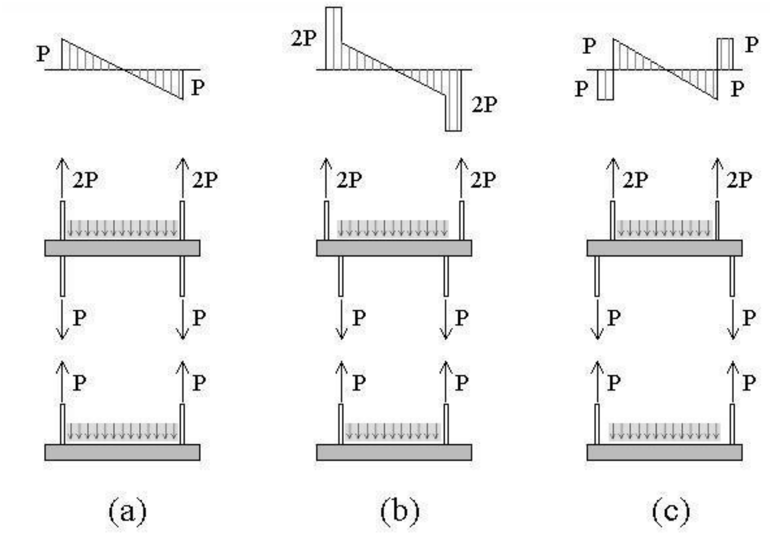

Figure 1.10: Shear loads on the transverse beam. (a) original design (continuous hanger bars); (b) offset design actually made; (c) offset design that would not have doubled the maximum shear loads.

Consequences

Several legal actions followed the Hyatt Regency Hotel Collapse. From several testimonies of experts in court, it emerged that the expression “standard practice” did not have an unequivocal meaning. As a consequence, in the following years several new regulations were issued, mainly aimed at:

- assessing more rigorously the responsibilities of the subjects involved in designing and building structures;
- improving communication between the subjects;
- making controls more effective, both on the design and on the built structures.

Nowadays, the Hotel is still in function. The walkways have been rebuilt, on columns.

Figure 1.11: One of the new walkways, on columns. (Source: www.flickr.com).

1.4.3 The Alfred P. Murrah Federal Building

On the 19th of April 1995, at 9:02 AM, a car bomb exploded in front of the Alfred P. Murrah Federal Building in Oklahoma City, Oklahoma, USA, causing the death of 168 people. According to the following investigations, about 80% of the victims were not directly due to the explosion, but to the following Progressive Collapse. It turned out that several design choices made the building very sensitive to Progressive Collapse. The results of the investigations prompted debates about building safety among American designers and politicians, and produced some changes in design regulations.

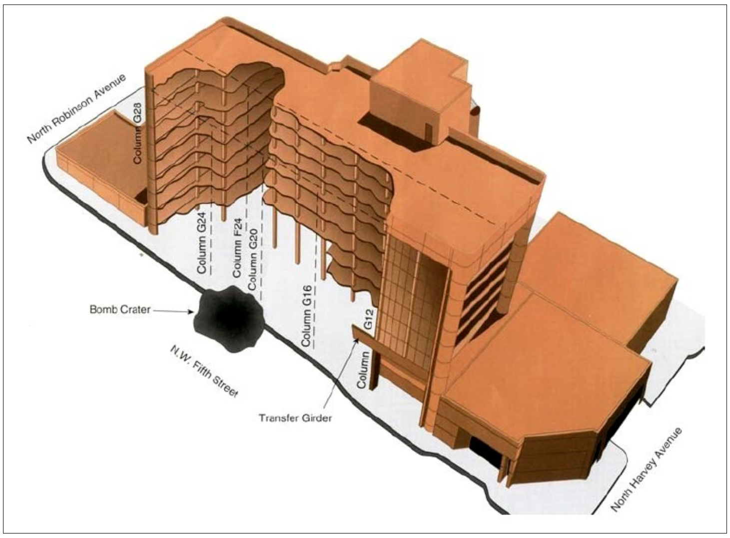

Figure 1.12: The A.P. Murrah Building in pristine state and after the bombing. (Source: NIST)

Description

Construction of the building lasted 20 months, between 1974 and 1976. The investigations recognized a good accordance between the blueprints and the actually built structure. The design consisted of 117 architectural and 40 structural drawings, which have been deemed very detailed and well made. Detail quality resulted significantly better than common practice for this type of buildings. Debris analyses found that the components were built according to the codes and that material resistances abundantly exceeded the minimum required by the design.
The used building code was the ACI 318-1971 "*Building Code Requirements for reinforced Concrete*"; at the time the code did not require to consider earthquakes, explosions or other extreme loads in Oklahoma City.

The Murrah Building consisted of a central nine-floor part with two single-floor wings and a partially underground parking lot. The structure of the nine-story part was cast-in-place concrete and had ten 6.1m long spans in one direction and two 10.7m spans in the other, plus shear walls and other resistant elements at the center of the south side (figure 1.13). Floor slabs were placed in the east-west direction.
One important element of the structural system was a transfer girder located at the third floor of the north facade. The girder supported intermediate columns, thus at the lowest two floors the spans of the north facade were 12.2m long (i.e., double as the other ones). The transfer girder had a rectangular 91x150cm section and was supported by rectangular 50x91cm columns. The facade at the two lower levels was re-entrant, so that there was a hollow volume under the third floor.

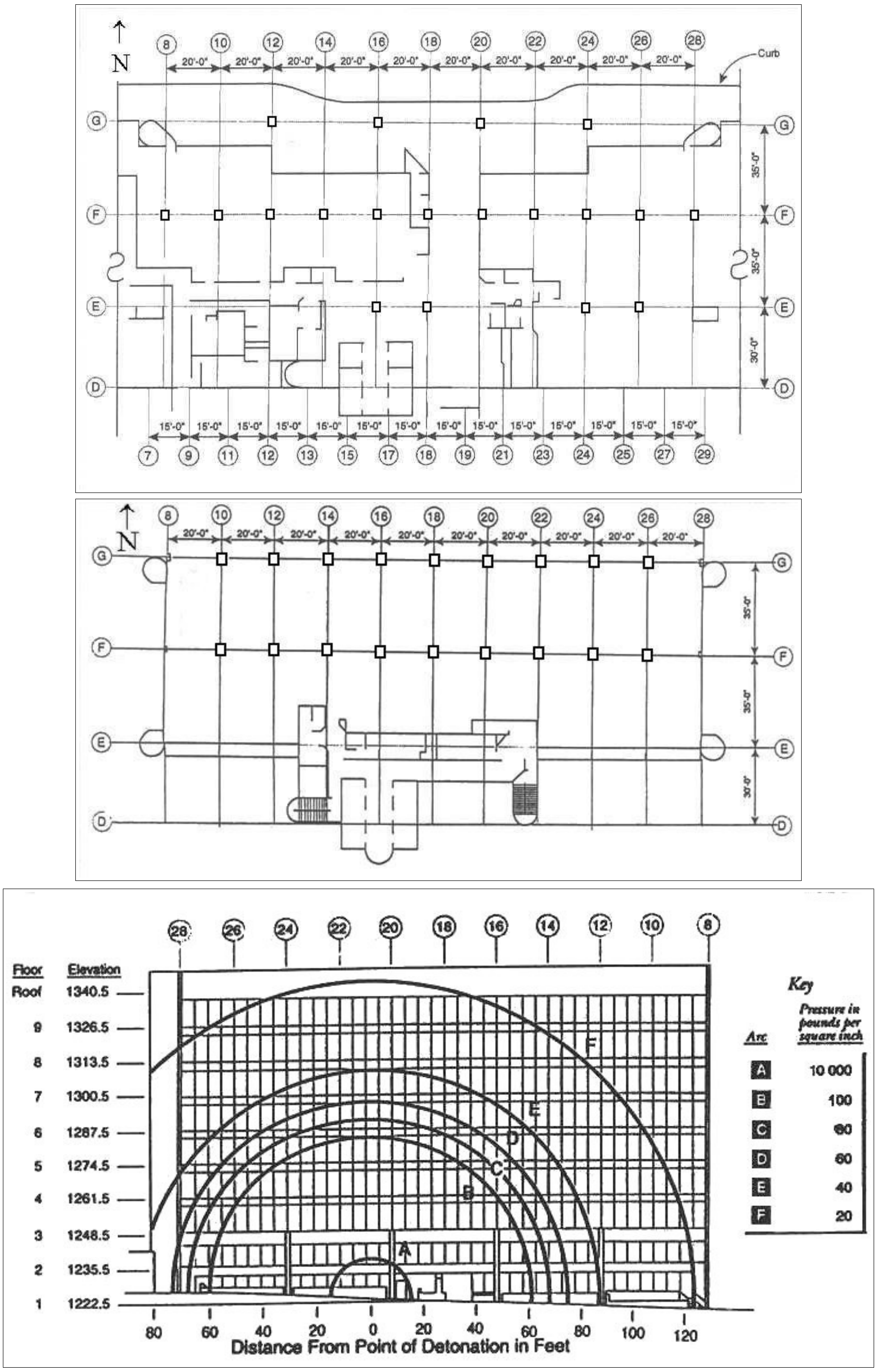

Figure 1.13: A.P. Murrah Building. (Top) Floor plan of the ground floor. (Middle) Floor plan of the upper floors. (Bottom) Front view of the north facade and peak overpressures of the bomb explosion. (Source: [11]).

Analysis

The north facade was directly hit by the explosion and was severely damaged. A big part of the north half of the building, comprised between vertical elements G12 and G28 (see figure 1.13 for the nomenclature) collapsed. Three columns (G16, G20 and G24) supporting the transfer girder were destroyed. This prompted the Collapse of the higher floors. Furthermore, between column lines 20 and 24 Collapse extended for the entire depth of the building. In all, about one half of the floor surface of the nine floor part collapsed; 112 of 180 floor slab panels between the second floor and the roof broke completely or partially. Damage to the facades not directly hit by the explosion was minimal. No significant global lateral or torsional displacements were noticed.

It has been deemed that the direct effect of the explosion was the destructions of three columns and some floor slabs, while the remaining Damage is deemed due to Progressive Collapse.
By studying the bomb crater and other produced Damages, it was estimated that the bomb had a potential equivalent to 4000lb (1814 kg) of TNT and was placed at a distance of about 4.75m from column G20. This column was abruptly removed by *brisance* (a shattering effect). This removed support of the transfer girder between columns G16 and G24. Analyses show that the structure, with this modification, would not have been not able to bear the vertical load of the floors above the third one. Furthermore, the explosion should have caused shear failure of columns G16 and G24. The loss of these columns leaves the transfer girder free of support from the east facade to column G12; of course, calculations show that this configuration is also not bearable by the structure.

The building lacked intrinsically redundancy and ductility. The weakest element was surely the transfer girder. It could not have been able to survive the removal of a column because its lower longitudinal reinforcement was not continuous at the columns (figure 1.14). Thus the girder was unable to withstand flexural moment reversal, forming de facto a simple hinge and overloading the adjacent extremities. Furthermore, all elements had in general much longitudinal reinforcement and little transverse reinforcement, giving them little ductility.

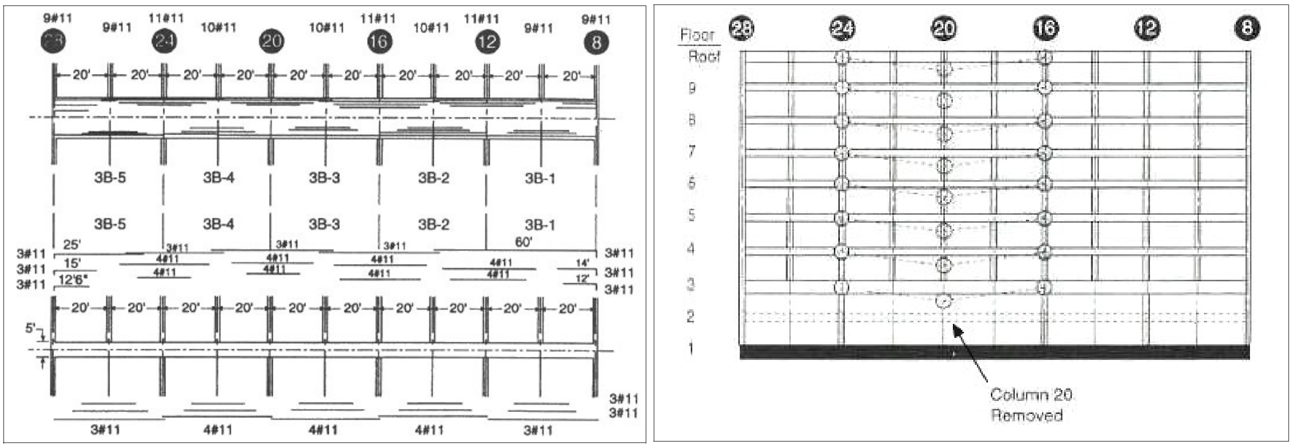

Figure 1.14: A.P. Murrah Building. (Left) Longitudinal reinforcement on the third floor transfer girder. It is to be noticed the absolute lack of continuity of the lower bars. (Right) Collapse mechanism of the north facade in case of removal of column G20 alone. (Source: [45]).

In 1985, about ten years after the building construction, the National Earthquake Hazards Reduction Program (NEHRP) issued the first recommendations aimed at making structures more capable of dissipating energy. It has been calculated that, if columns G16 and G24 were made according to the new recommendations, they would have had a high probability of surviving the explosion without shear failure. Because of the proximity to the bomb, column G20 would probably have been shattered even if it was made according to the recommendations.

It has been estimated that, if the seismic recommendations had been adopted, the following scenarios could have happened:
1. if column G20 resisted the explosion, then the structural Damage would have been limited to the floor slabs directly destroyed by the explosion. The surface losses would have been reduced

by 85%
2. if only column G20 was removed by the explosion and the transfer girder had resisted, the global structural losses would have been reduced by 80%;
3. if column G20 was destroyed and the transfer girder was non able to bear the span between G16 and G24, then the structural losses would have been the elements directly hit by the explosion and the floor panels between column lines F and G, 16 and 24. The surface loss would have been reduced by 50%.

Consequences
Oklahoma City was the first attack against a federal building inside the USA; at the time this eventuality had never been considered. There were recommendations for government buildings built abroad (*"Structural Engineering Guidelines for New Embassy Office Buildings*", published by the Department of State), and the first move was to extend these recommendations to the new federal buildings built inside the USA. The main recommendations were about the minimum standoff distance of vehicles, ductility and steel reinforcement continuity.
After the building was built several regulations on constructions in seismic areas had been issued, which provide for a high level of ductility. Several studies proposed that prescriptions for structural details in seismic areas should also be applied to new buildings in non seismic areas, disregarding of seismic lateral forces. For existing buildings it was proposed to adopt techniques of seismic improvement like the addition of structural walls.

The first official document issued after the Oklahoma City bombing was the *Proceedings of the 1996 ASCE Structure Congress,* which includes suggestions and references to design structures potentially subjected to terroristic attacks. The document consists in chapters that correspond to design steps:
- the first step is to quantify the level of risk of the design. To achieve this, an algorithm is given;
- the second step is the quantification of parameters of the loads associated with a particular risk. For example, the interesting parameters for explosions are the overpressure peak, the length and the shape of the pressure wave;
- then a structural system that withstands the calculated loads is chosen. One main decision criterion is the ability to provide ductility and redundancy;
- then the single structural elements are designed for the dynamic loads of the design explosion. According to the document, design can be effectively made with models of Single Degree Of Freedom (SDOF) oscillators. The analysis is generally performed with non-factorized parameters (i.e. not multiplied by safety coefficients) and provides for inelastic response and damaging of the structural elements.

It must also be observed that a debate arose, about the costs of applying the suggestions. The difference in cost between a normal building and one with anti-seismic details has been estimated in 1-2%, and with one with the specific Collapse analysis has been estimated in 5-7%. Since thousands of federal buildings exist, the debate was mainly based on the question if the extra costs involved are justified by the social costs of a possible Collapse.

1.4.4 The World Trade Center

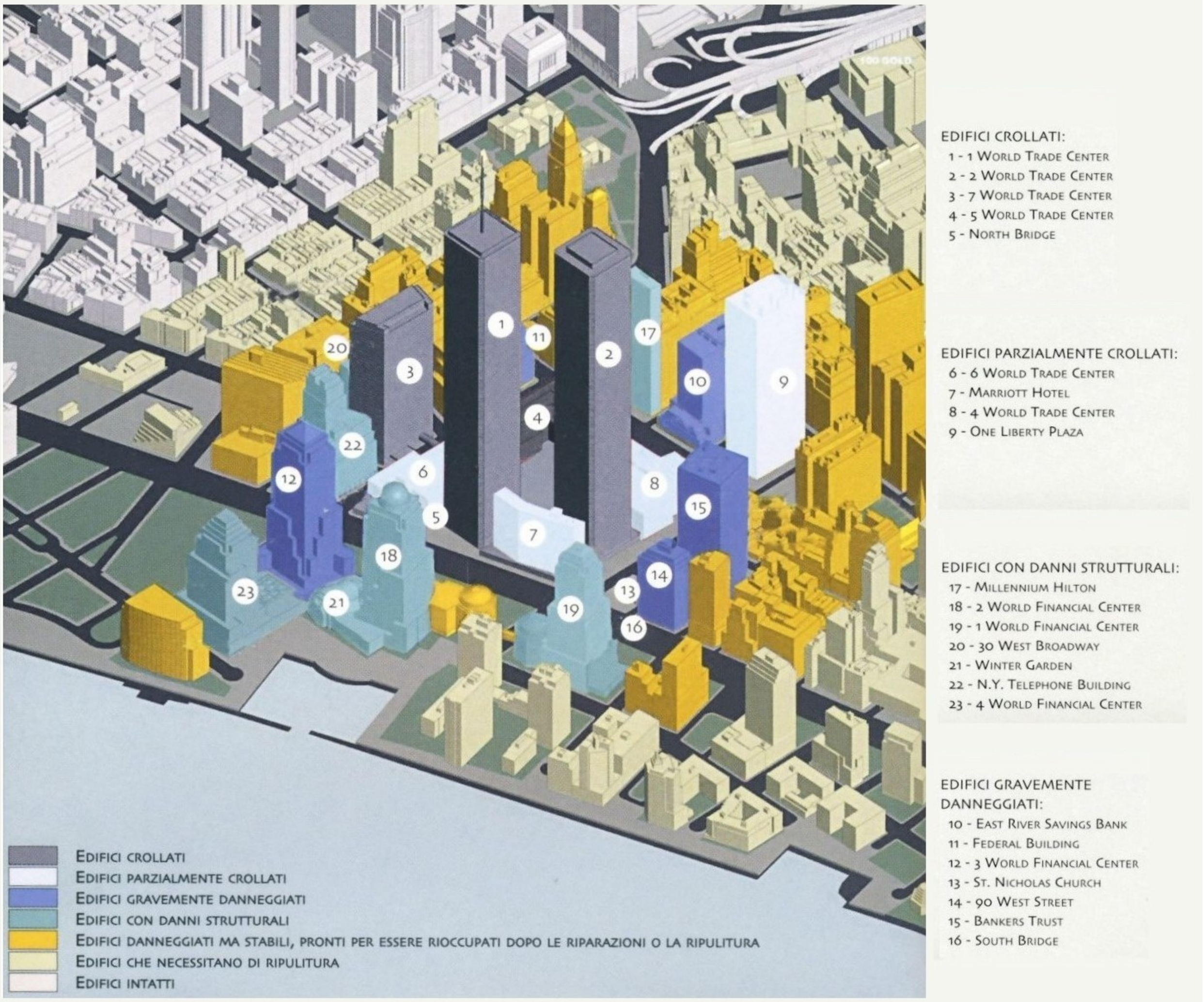

Figure 1.15: Aerial view of the World Trade Center complex and its surrounding buildings. Each color corresponds to the amount of Damage suffered on September 11th, 2001. (Source: [44]).

1.4.4.1 Buildings 1 and 2 of the World Trade Center

Description

The World Trade Center (WTC) was a a complex of seven buildings located in the southern part of Manhattan island (New York city, USA). It comprised two 110-floor office buildings (WTC1 and WTC2), a 22-floor hotel (WTC3), two 9-floor office buildings (WTC4 and WTC5), one 8-floor office building (WTC6) and one 47-floor office building (WTC7). It was property of the Port Authority of New York; its construction required an estimated cost of 1.29 billion US dollars (adjusted to 1992).

Construction of the structures started in 1968. WTC1 (North Tower) and WTC2 (South Tower) started to be occupied in 1970 and 1972, respectively, even though the official inauguration of the Center was in 1973.

The North Tower was about 416m tall and supported a 110m long antenna; the South Tower was about 414m tall. Each Tower had a square floor plan with sides of about 64m and beveled corners. They included 110 floors in elevation plus 6 underground floors.

The structures consisted of four major structural subsystems: a central core, the exterior wall, the floor system and the “hat truss”.

- The central core consisted in 47 steel columns, which extended virtually the full length of the building, interconnected by a grid of steel beams. In the floor plans it corresponded to the central rectangular area, about 41m by 26m, in which stairs, elevators and bathrooms were concentrated.
- The exterior wall was a square tube, consisting of 236 narrow steel columns connected by spandrels, located in the facades. It consisted of prefabricated welded modules, three stories tall and three columns wide, bolted to the adjacent units. At the lower floors, bundles of three columns merged together to allow transit.
- The floor system consisted of lightweight concrete on a steel deck, supported by a grid of steel trusses. The trusses covered spans of 11m or 18m. One end of each truss had a viscoelastic device to dampen horizontal oscillations. The floor system did not only support vertical loads; it also had the important function of connecting the central core and the exterior walls, giving a high horizontal stiffness to the tower. Furthermore, the lack of structural elements between the core and the facades provided for great flexibility in the internal partitions.
- The “hat truss” was a set of steel braces, located from the 107th floor to the roof of each tower. Its primary purpose was to support a tall antenna atop each tower, although only WTC1 had one installed. The hat truss provided additional connections among the columns, providing additional means for load redistribution.

Ten different grades of steel, with yield strength ranging from 36ksi to 100ksi were used for the structural elements. For fireproofing, most of the core columns were protected by sheets of gypsum wallboard. The other elements were coated with sprayed fire-resistive materials (the elements of the exterior wall were also enclosed in a sheet-aluminum cover).

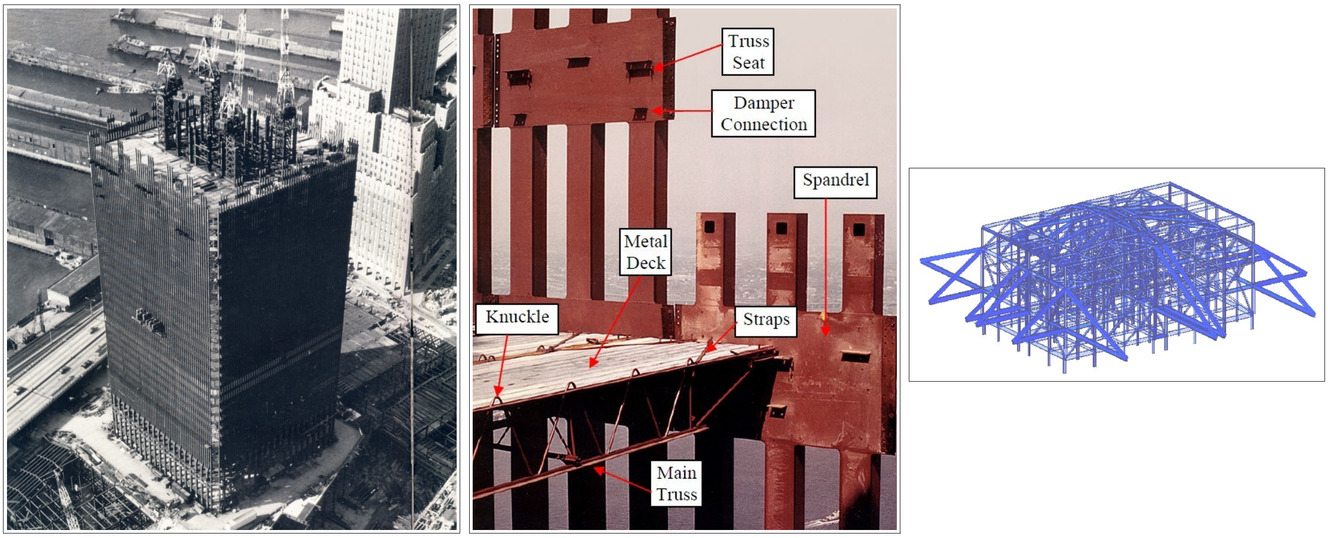

Figure 1.16: (Left) The North Tower (WTC1) under construction. Central core, exterior wall and floor system are clearly distinguishable. (Center) Detail of the connection between floor system and exterior wall during assembly. It is worth noticing the staggering of the prefabricated facade modules. (Right) Representation of the hat truss. (Sources: [44], [34]).

Analysis
On the 11th of September 2001, at 8:46 AM, the North Tower (WTC1) was hit by an aircraft. The building collapsed 102 minutes later, at 10:28 AM. The South Tower (WTC2) was hit by another aircraft at 9:03 AM and collapsed 56 minutes later, at 9:59 AM.

In WTC1, the most of the impact Damage was confined to the 95th and 96th floors. Summed over all floors, the estimated Damages are: 35 exterior columns severed and 2 heavily damaged; 6 core columns severed and 3 heavily damaged; 43 (out of 47) core columns stripped of insulation on one or more floors; insulation stripped from trusses covering 60'000ft^2 ($\approx$5500m^2) of floor area.
In WTC2, the bulk of the impact Damage was confined to six floors (78 to 83). Summed over all floors, the estimated Damages are: 33 exterior columns severed and 1 heavily damaged; 10 core columns severed and 1 heavily damaged; 39 (out of 47) core columns stripped of insulation on one or more floors; insulation stripped from trusses covering 80'000ft^2 ($\approx$7500m^2) of floor area.
In both cases, a big amount of jet fuel spilled in the buildings and fed fires.

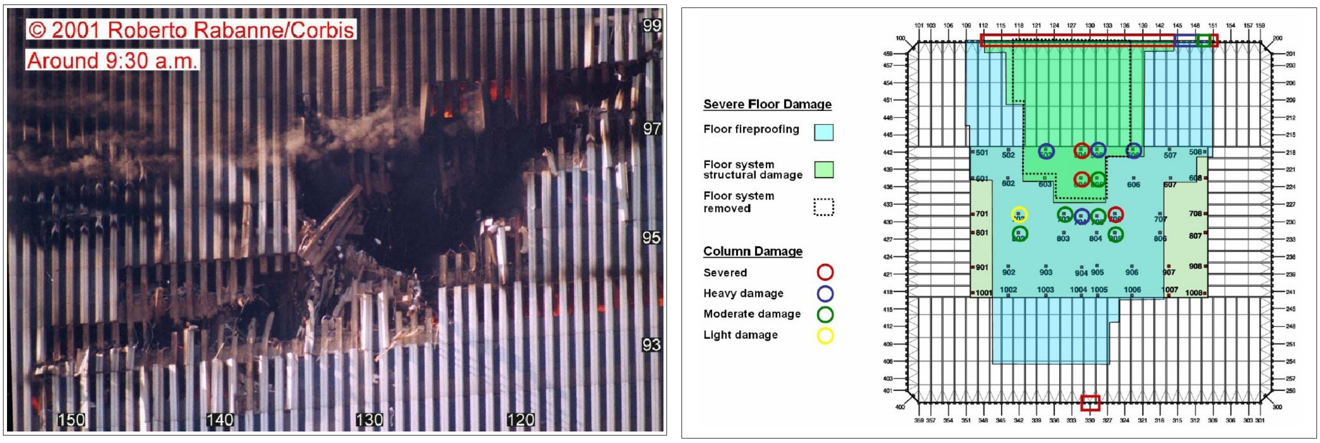

Figure 1.17: (Left) Impact Damage observed on the north face of WTC1. (Right) One estimated impact Damage scenario of WTC1. (Source: [34]).

According to the *Final Report on the Collapse of the World trade Center Towers* [34], the probable Collapse sequence for both towers was the following:
- each aircraft impact severed exterior columns, damaged interior core columns and knocked off insulation from steel structural elements. The weight carried by the severed columns was distributed to other columns.
- Subsequently, fires began to grow and spread. Because of these these fires, the building core weakened and began losing its ability to carry loads.
- The floors weakened and sagged from the fires, pulling inward on the exterior columns.
- Floor sagging and exposure to high temperatures caused the exterior columns to bow inward and buckle.
- Collapse then ensued.

Furthermore, according to the *Final Report* [34] the WTC towers likely would not have collapsed under the combined effects of aircraft impact Damage and the extensive fires that followed, if the thermal insulation had not been widely dislodged or had been only minimally dislodged by the impacts. In other words, the towers had enough redundancy to withstand a considerable initial Damage, in absence of the effects of fire.

The Bažant study
A few hours after the Collapses, Prof. Zednek P. Bažant of the Northwestern University, Evanston (Illinois, USA) released a study of the Collapse mechanism [7]. The study highlights that, once the upper part of each building started to fall down and hit the lower part, arresting the Collapse was impossible because the dynamic load was much bigger than the available bearing capacity.

To reach these conclusions, first the Author estimates the elastic overload ratio of the falling part of the structure on the lower part. This is achieved with very simplified calculations, using two different methods.

In the first method the Author assimilates the lower part of the tower to a Single Degree Of Freedom (SDOF) oscillator. All the reported parameters refer to the North Tower. The stiffness of the oscillator is estimated in the most optimistic case, i.e. with all undamaged columns and with all loads evenly distributed. The estimated stiffness of the oscillator is C≈71 GN/m (1 GN=10^9 N; the Author does not specify on which bases this value is obtained). The estimated mass of the upper part of the North Tower is m≈58·10^6 kg. By equating the loss of gravitational potential energy of the falling (upper) part in a one floor fall and the deformation energy of the lower part at maximum elastic deflection, the overload ratio due to impact of the upper part is calculated as

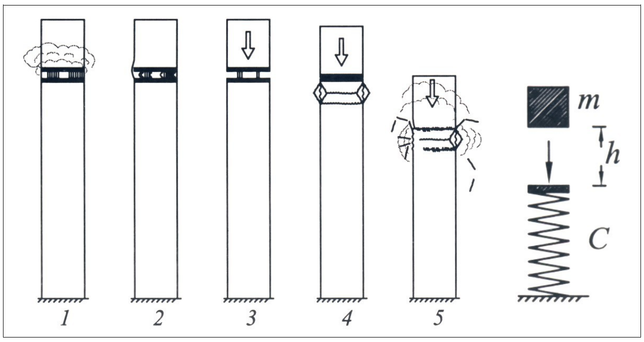

Figure 1.18: Schematic representation of the Collapse sequence and equivalent SDOF oscillator of the Bažant study. (Source: [7]).

$$\frac{P_{dyn}}{P_0} = 1 + \sqrt{1 + (2Ch/mg)} \approx 31$$

in which:
h≈3.7m is the falling height of the mass, which is equal to the height of critical floor columns;
g is the gravity acceleration;
P_{dyn} is is the static force that produces maximum deflection;
P_0=mg is the weight of the upper part of the building.

The other estimate is obtained with the following formula, which is derived from an elastic wave equation, according to a theory reported by the Author in one of his books:

$$\frac{P_{dyn}}{P_0} = (A/P_0)\sqrt{2\rho g E_{ef} h} \approx 64.5$$

in which
A is the cross section of the building;
E_{ef} is the stiffness of the cross sections of all columns, divided by A;
ρ is the mass of the building for volume unit.

Because of the approximations, these two results are just indicative. It is meaningful that according to both calculations the falling of the upper part produces effects of a higher order of magnitude than the static force.

Subsequently, the Author tries to find out if the fall of the upper part can be arrested by energy dissipation during plastic buckling which follows the initial elastic deformation.

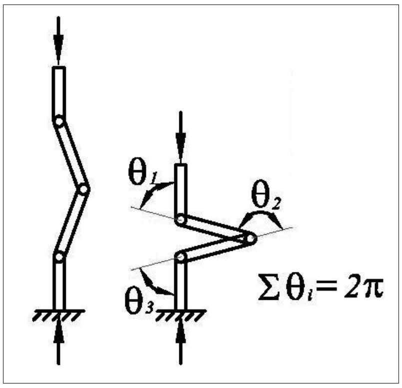

Figure 1.19: Schematic representation of a buckled column in the Bažant study. (Source: [7]).

To do it, it is assumed that every column buckles and three plastic hinges form in each one (figure 1.19). Assuming that each plastic hinge can rotate indefinitely without breaking, the sum of their rotations cannot be higher than the combined rotation angle $\Sigma\theta_i=2\pi$.
By calculating the maximum plastic deformation energy and multiplying it by the number of columns, the optimistic estimate W_p=0.5GNm is obtained.
In order to have the maximum combined rotation angle $\Sigma\theta_i=2\pi$, the upper part of the building must fall one whole floor height. Thus the variation of the gravitational potential energy between the beginning of the fall and the maximum plastic deformation of the columns is $W_g=2hmg\approx 4.2$GNm.
In conclusion, the ratio between the energy to dissipate and the energy that it is possible to dissipate is $W_g/W_p\approx 8.4$. This estimate is highly optimistic, because in reality the plastic hinges would break for rotations much smaller than the ones considered, and because the actual falling height was likely 3 to 10 floors (which would make the estimated ratio 3 to 10 times higher).

Other cases of impact of aircrafts on high-rise buildings
It must be highlighted that other impacts of aircrafts on high-rise buildings have happened, but in these cases the buildings did not collapse. The following is a list of reported cases:

- On July 28, 1945, a US Army B-25 bomber crashed into New York's 102-story tall Empire State Building, killing 3 crew members and 11 other people. Low visibility due to fog was identified as the main cause of the crash.
- On May 20, 1946, a US Army C-45 Beechcraft airplane crashed into New York's 70-story tall 40 Wall Street Building (known today as the Trump Building), killing 5 who were on the plane. Again, low visibility due to fog was identified as the main cause of the crash.
- On January 5, 2002, a 15-year-old boy stole a Cessna 172 airplane from a flight school and committed suicide by crashing into the 42-story tall Bank of America Tower in Tampa, Florida, USA. He was the only fatality.
- On April 18, 2002, a Rockwell Commander 112 airplane crashed into the 31-story tall Pirelli Tower in Milan, Italy, killing the pilot, who was alone in the airplane, and 2 occupants of the building. It is unclear if it was an accident or if the pilot committed suicide.
- On October 11, 2006, a Cirrus SR20 light aircraft crashed into the 50-story tall Belaire Apartments building in New York, killing the 2 people who were on the plane. Pilot error was identified as the cause of the crash.

High rise buildings have necessarily a very redundant structure, because of the high loads that they must bear. Not only the vertical loads must be borne; the horizontal loads of wind and earthquake are very important. Furthermore, the big number of occupants and the relatively scarcity of escape paths leads to the use of bigger safety coefficients than normal buildings.
The wind load is especially critical, because the big surface of the facades makes the total force extremely strong. The Twin Towers were designed to withstand winds up to 225km/h.
The Twin Towers were also designed to withstand an impact with a Boeing 707-320, which was the biggest aircraft in existence when the WTC was designed. Compared to this model, the Boeing

767-200 aircrafts that hit the Twin Towers have a maximum full load weight about 15% higher. According to Bažant [7], this difference is well within the safety margins of the design.

The best documented case of aircraft impact on a high rise building happened on the 28th of July 1945, when a B52 bomber collided on the 79th floor of the Empire State Building, in New York city. The impact provoked a hole 5.5m wide and 6m high in the building's facade. The consequent fire remained confined. All the 14 victims were either on the plane or in areas close to the impact point. The death toll could have been worse because both engines separated from the aircraft and kept moving, in flames, by inertia.
The aircraft impacted exactly the 79th floor system, and this contributed to distributing its thrust in the structure. The numerous steel columns, located at a mutual distance of about 5.8m, give the building a great redundancy. The structure did not suffer Damage other than in the impact area. In fact, it has been calculated that the global thrust of the impact was about two hundred times lower than the design wind thrust. The mass of the aircraft was about 10^4kg, as opposed to $8x10^7$kg of the building. The occupants that were in the building, but distant from the impact area, referred to have felt a “light tremor” of the structure and an oscillation that did not last long (the oscillation was probably dampened rapidly by the masonry wall panels).

Figure 1.20: The Empire State Building after the 1945 aircraft collision and a detail of the impact zone. (Source: unknown).

1.4.4.2 Building 7 of the World Trade Center

Description

Building 7 of the World Trade Center was a 47 story office building located immediately to the north of the main WTC complex, approximately 105m from the north side of WTC 1 (figure 1.21). The floor plan of WTC 7 was an irregular trapezoid, approximately 100m long on the north face and 75m long on the south face, 44m wide. The 186m tall building contained approximately 200'000 m^2 of floor area.

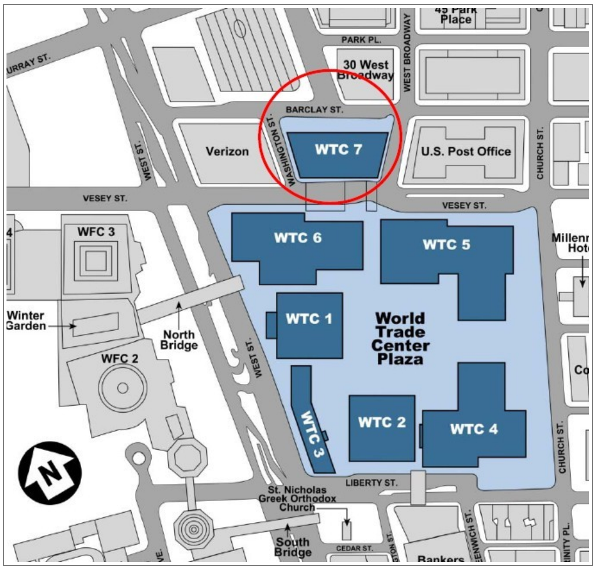

Figure 1.21: Location of WTC7. (Source: NIST).

The structure was steel frame. From the 7th floor to the 47th floor, WTC7 was supported by 24 interior columns and 58 perimeter columns (figure 1.22). Twenty-one of the interior columns formed a rectangular building core. The remaining three interior columns (labeled 79, 80, and 81) were particularly large, as they provided support for the long floor spans on the east side of the building.

The lower floors had a different column arrangement. Therefore, a set of column transfers were constructed within the 5th and 7th floor slabs.
The floor structures were composed of reinforced concrete of varying thickness on top of corrugated metal decking. The floor beams were connected to the concrete deck by shear studs to allow composite action of the concrete slab and the steel elements. The floor beams were connected to girders, which framed into the columns.
Sprayed fire-resistive material was applied to the structural steel and to the underside of the metal floor decking. Active fire protection systems comprised fire sensors and alarms, notification systems, automatic fire sprinklers, water supplies, and smoke management.

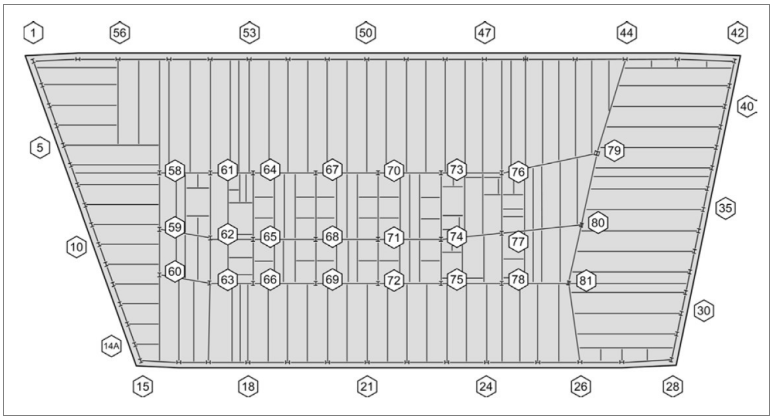

Figure 1.22: Typical floor plan of WTC7, from the 7th to the 47th story. (Source [36]).

Analysis
According to the *Final Report on the Collapse of World Trade Center Building 7* [36], none of the large pieces of debris from WTC2 hit WTC7 because of the large distance between the two buildings. Pieces of WTC1 hit WTC7, severing six columns on Floors 7 through 17 on the south face and one column on the west face near the southwest corner. The debris also caused structural Damage between Floor 44 and the roof.
Most likely, the WTC7 fires began as a result of burning debris from the Collapse of WTC1. Unlike the Towers, where dispersion of jet fuel caused simultaneous fire initiation over extensive areas and multiple adjacent floors, in WTC7 were typically observed multiple single floor fires that started in local origins and that were fed by typical office combustibles, such as furniture and appliances. Fires were observed spreading on the 7th floor through the 13th floor, with the exception of the 10th floor. On some floors the fires were limited by automatic sprinklers, whose water came from the storage tanks on the 47th floor. However, on the lowest 20 floors the sprinkler system was not working because it relied on the city water system, which had been damaged by the Collapse of the Towers.

According to the *Final Report* [36], the sequence that most likely led to the Collapse of WTC7 is the following:

- WTC7 endured fires for almost seven hours before Collapse happened (from 10:28AM until 5:21PM). Prolonged heating of the long beams resulted in proportionately large thermal elongation relative to the other components of the floor system, compressing the beams along their length. This led to distortion of the beams and breaking of the connections between beams and floor slabs.
- Some floor beams expanded enough to push the girder spanning between Columns 79 and 44 out of its support at Column 79. The unsupported girder and other local fire-induced Damage caused floor 13 to collapse, beginning a cascade of floor failures down to the 5th floor (which was much thicker and stronger than the other floors). This left column 79 without lateral support for nine floors; as a consequence, the column buckled, becoming the initial local failure for Collapse initiation.

– The buckling of column 79 led to a vertical progression of floor failures up to the roof and to the buckling of columns 80 and 81. An east-to-west horizontal progression of interior column buckling followed. As the failed building core moved downward, its loads were redistributed to the exterior columns, which subsequently buckled. Global Collapse occurred as the entire building above the buckled region moved downward as a single unit.

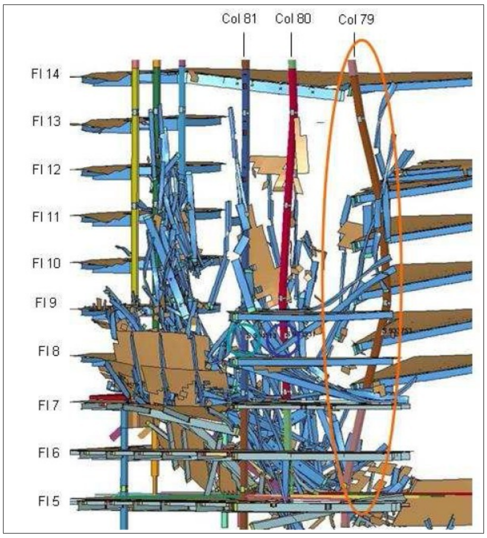

Figure 1.23: Model of the WTC7 Collapse. Still frame of the area where the Collapse started, just after column 79 buckled. (Source: [36]).

According to the *Final Report* [36], "*The collapse of WTC7 represents the first known instance of the total collapse of a tall building primarily due to fires*", and WTC7 would have collapsed from the fires even without the initial structural Damage caused by debris impact. Furthermore, removal of a section of column 79 between Floors 11 and 13 would have led to the Collapse of the entire building, even in absence of debris impact and fire. Yet no evidence was found to suggest that WTC7 was not designed in a manner generally consistent with building codes in effect at the time of construction, and that it was adequately designed for vertical loads due to gravity and lateral loads due to wind. The structural design did not explicitly evaluate fire effects, which was typical for engineering practice at that time the WTC was designed (as well as at the time the *Final Report* [36] was published).

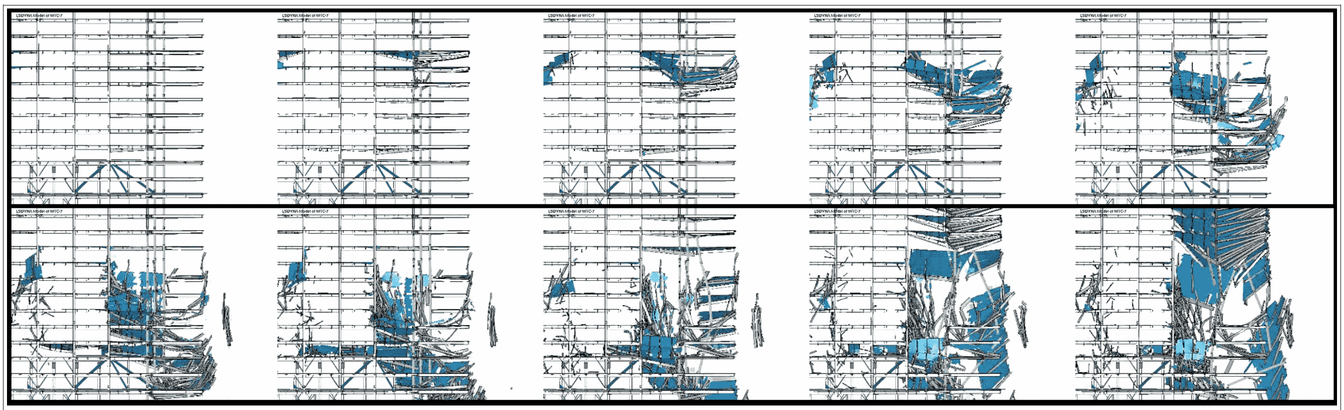

Figure 1.24: Model of the WTC7 Collapse. Sequence of the start of the Collapse. (Source: NIST).

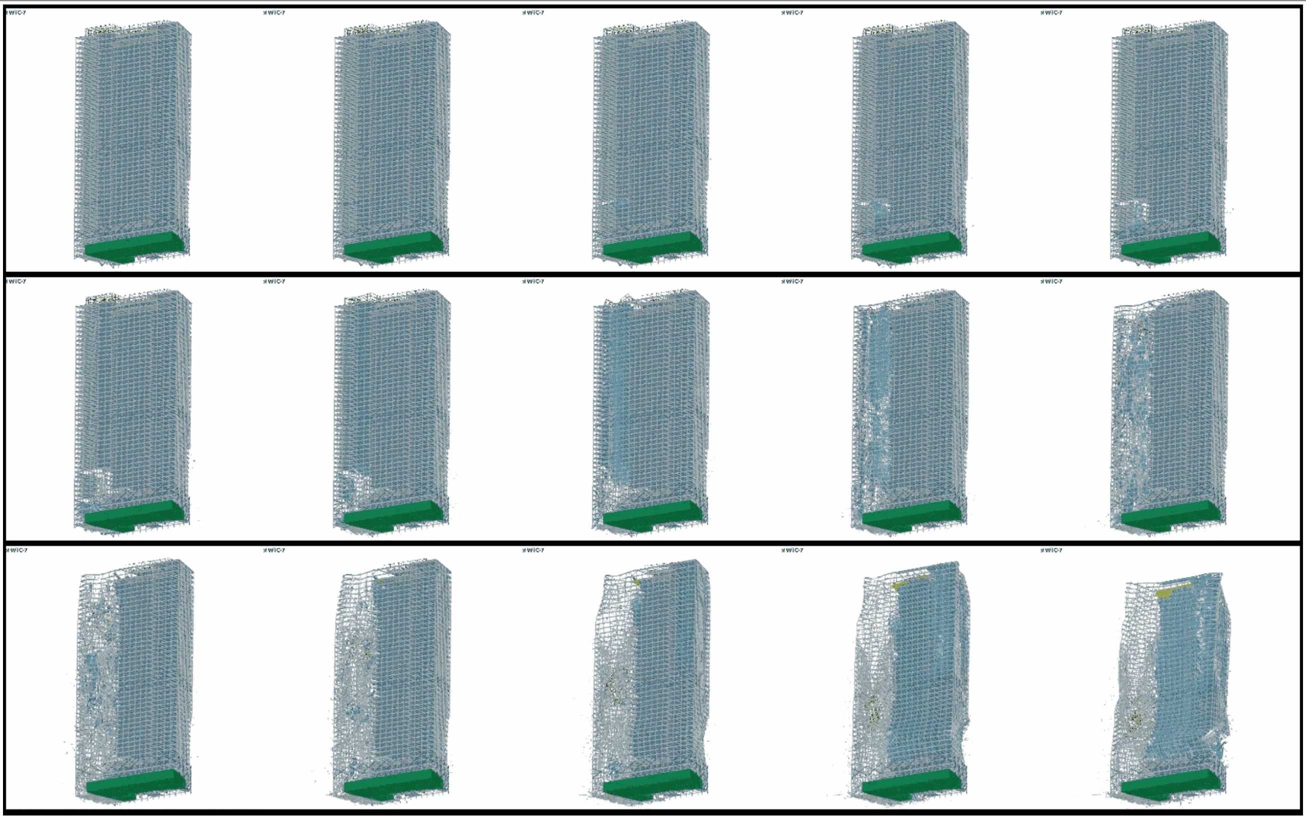

Figure 1.25: Model of the WTC7 Collapse. Sequence of the whole building Collapse. (Source: NIST).

Consequences of the WTC Collapses

The WTC Collapses had a strong impact on the public opinion, on the technical community and on the political world. In particular, they gave they gave new impulse to research and regulations on Progressive Collapse mitigation, as the following chapters show.

Immediately after the WTC attacks, the Federal Emergency Management Agency (FEMA) and the American Society of Civil Engineers (ASCE) did a building performance study of the disaster "*to determine probable failure mechanisms and to identify areas of future investigation that could lead to practical measures for improving the damage resistance of buildings against such unforeseen events*" [34]. The Building Performance Study Team issued its report in May 2002. The study team consisted of experts who largely volunteered their time away from their other professional commitments.

A much deeper investigation was conducted by the National Institute of Standards and Technology (NIST). The investigation was announced on August 21, 2002, with funding from the US Congress. On October 1, 2002, the Congress signed a law ("National Construction Safety Team Act") under which authority the the NIST investigation was conducted.

The goals of the investigation of the WTC disaster were:

"To investigate the building construction, the materials used, and the technical conditions that contributed to the outcome of the WTC disaster.

To serve as the basis for:

- *Improvements in the way buildings are designed, constructed, maintained, and used;*
- *Improved tools and guidance for industry and safety officials;*

– *Recommended revisions to current codes, standards, and practices; and*
– *Improved public safety.*

The specific objectives were:
1. *Determine why and how WTC1 and WTC2 collapsed following the initial impacts of the aircraft and why and how WTC7 collapsed;*
2. *Determine why the injuries and fatalities were so high or low depending on location, including all technical aspects of fire protection, occupant behavior, evacuation, and emergency response;*
3. *Determine what procedures and practices were used in the design, construction, operation, and maintenance of WTC1, WTC2, and WTC7; and*
4. *Identify, as specifically as possible, areas in current building and fire codes, standards, and practices that warrant revision."*

A staff of over 200 contributed to the investigations. The investigation team accumulated over 300 hours of video footage and more than 7000 photographs; conducted interviews with over 1000 people; analyzed hundreds of steel pieces obtained from the wreckage; performed laboratory tests. Computer simulations of the sequence of events that happened from the aircraft impacts to the initiation of Collapse were performed for WTC1 and WTC2. For WTC7, the Collapse sequence was also modeled.
During the course of the investigation, public briefings and meetings were held to solicit input from the public, present preliminary findings, and obtain comments on the direction and progress of the investigation.

Eventually, the *Final Report on the Collapse of the World Trade Center Towers* [34] was issued in September 2005, while the *Final Report on the Collapse of World Trade Center Building 7* [36] was issued in November 2008.
Other than the already mentioned conclusions about the sequence of events, the Reports include a series of recommendations for increased structural integrity, enhanced fire endurance of structures, new methods for fire resistant design of structures, enhanced active fire protection, improved building evacuation, improved emergency response, improved procedures and practices, and education and training. The first Report includes 30 recommendations; the second Report includes 13, of which 12 are reiterated and 1 is new. Two recommendations regard Progressive Collapse, and are here reported.

The first one is called "Recommendation 1" in the Twin Towers Report and "Recommendation A" in the WTC7 Report.
NIST recommends that: (1) progressive collapse be prevented in buildings through the development and nationwide adoption of consensus standards and code provisions, along with the tools and guidelines needed for their use in practice; and (2) a standard methodology be developed—supported by analytical design tools and practical design guidance—to reliably predict the potential for complex failures in structural systems subjected to multiple hazards.

The Twin Towers Report also elaborates:
a. Progressive collapse should be prevented in buildings. The primary structural systems should provide alternate paths for carrying loads in case certain components fail (e.g., transfer girders or columns). This is especially important in buildings where structural components (e.g., columns, girders) support unusually large floor areas. Progressive collapse is addressed only in a very limited way in practice and by codes and standards. For example, the initiating event in design to prevent progressive collapse may be removal of one or two columns at the bottom of the structure. Initiating events at multiple locations within the structure, or involving other key components and

subsystems, should be analyzed commensurate with the risks considered in the design. The effectiveness of mitigation approaches involving new system and subsystem design concepts should be evaluated with conventional approaches based on indirect design (continuity, strength, and ductility of connections), direct design (local hardening), and redundant (alternate) load paths. The capability to prevent progressive collapse due to abnormal loads should include: (i) comprehensive design rules and practice guides; (ii) evaluation criteria, methodology, and tools for assessing the vulnerability of structures to progressive collapse; (iii) performance-based criteria for abnormal loads and load combinations; (iv) analytical tools to predict potential collapse mechanisms; and (v) computer models and analysis procedures for use in routine design practice. The federal government should coordinate the existing programs that address this need: those in the Department of Defense; the General Services Administration; the Defense Threat Reduction Agency; and NIST. Affected Standards21: ASCE-7, AISC Specifications, and ACI 318. These standards and other relevant committees should draw on expertise from ASCE/SFPE 29 for issues concerning progressive collapse under fire conditions. Model BuildingCodes: The consensus standards should be adopted in model building codes (i.e., the International Building Code and NFPA 5000) by mandatory reference to, or incorporation of, the latest edition of the standard. State and local jurisdictions should adopt and enforce the improved model building codes and national standards based on all 30 WTC recommendations. The codes and standards may vary from the WTC recommendations, but satisfy their intent. b. A robust, integrated predictive capability should be developed, validated, and maintained to routinely assess the vulnerability of whole structures to the effects of credible hazards. This capability to evaluate the performance and reserve capacity of structures does not exist and is a significant cause for concern. This capability also would assist in investigations of building failure—as demonstrated by the analyses of the WTC building collapses carried out in this Investigation. The failure analysis capability should include all possible complex failure phenomena that may occur under multiple hazards (e.g., bomb blasts, fires, impacts, gas explosions, earthquakes, and hurricane winds), experimentally validated models, and robust tools for routine analysis to predict such failures and their consequences. This capability should be developed via a coordinated effort involving federal, private sector, and academic research organizations in close partnership with practicing engineers.

The second recommendation is called "Recommendation B" and is only present in the WTC7 Report.
NIST recommends that buildings be explicitly evaluated to ensure the adequate performance of the structural system under maximum credible (infrequent) design fires with any active fire protection system rendered ineffective. Of particular concern are the effects of thermal expansion in buildings with one or more of the following features:
(1) long-span floor systems which experience significant thermal expansion and sagging effects,
(2) connection designs (especially shear connections) that cannot accommodate thermal effects,
(3) floor framing that induces asymmetric thermally-induced (i.e., net lateral) forces on girders,
(4) shear studs that could fail due to differential thermal expansion in composite floor systems, and (5) lack of shear studs on girders. Careful consideration should also be given to the possibility of other design features that may adversely affect the performance of the structural system under fire conditions.

Building owners, operators, and designers are strongly urged to act upon this recommendation. Engineers should be able to design cost-effective fixes to address any areas of concern that are identified by these evaluations. Several existing, emerging, or even anticipated capabilities could have helped prevent the collapse of WTC 7. The degree to which these capabilities improve performance remains to be evaluated. Possible options for developing cost-effective fixes include:
• More robust connections and framing systems to better resist the effects of thermal expansion on

the structural system.
• Structural systems expressly designed to prevent progressive collapse. The current model building codes do not require that buildings be designed to resist progressive collapse.
• Better thermal insulation (i.e., reduced conductivity and/or increased thickness) to limit heating of structural steel and to minimize both thermal expansion and weakening effects. Currently, insulation is used to protect steel strength, but it could also be used to maintain a lower temperature in the steel framing to limit thermal expansion.
• Improved compartmentation in tenant areas to limit the spread of fires.
• Thermally resistant window assemblies which limit breakage, reduce air supply, and retard fire growth.
Industry should partner with the research community to fill critical gaps in knowledge about how structures perform in real fires, particularly considering: the effects of fire on the entire structural system; the interactions between subsystems, elements, and connections; and scaling of fire test results to full-scale structures, especially for structures with long span floor systems.

1.4.5 The Eads Bridge

The Eads Bridge is a railway bridge that crosses the Mississippi river in St.Louis, Missouri, USA. Its construction was completed in 1874. It consists of three steel truss arches that cover a total length of about 500m. This structure is particular for its great redundancy; this is not casual, as it was conceived like this in its design phase. At the inauguration of the bridge, its designer James Buchanan Eads stated: "*The peculiar construction of the superstructure is such that any piece in it can be easily taken out and examined, and replaced or renewed, without interrupting the traffic of the bridge... In completing the western span two of the lower tubes of the inside ribs near the middle of the span were injured during erection, and were actually uncoupled and taken out without any difficulty whatever, after the span was completed, and two new ones put in their place in a few hours*" [35].
In October 1969 a tug boat knocked out a section of one of the arches, but the Damage remained limited to the hit elements.

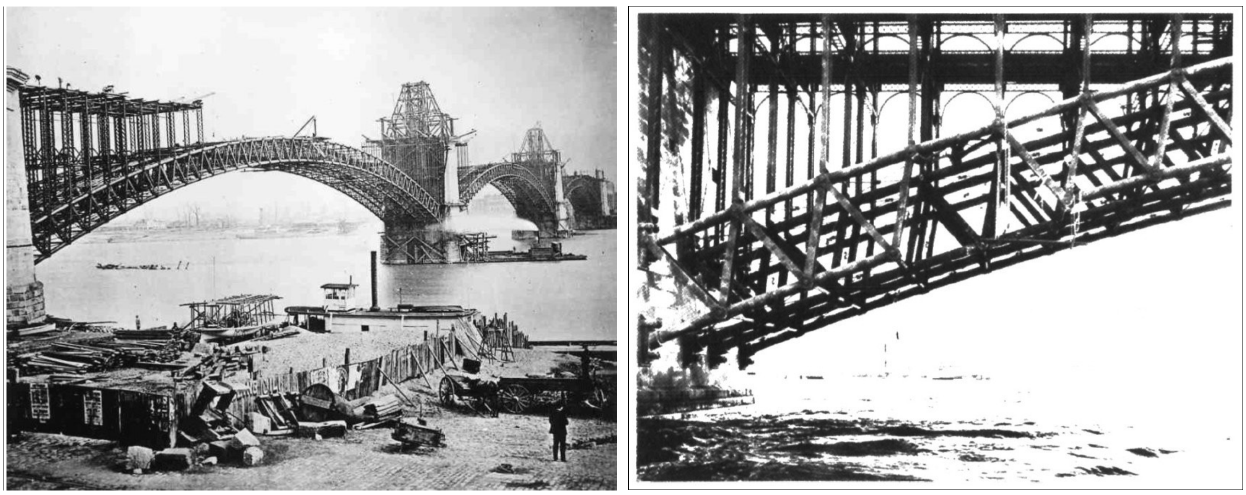

Figure 1.26: The Eads Bridge during construction and detail of the Damage it suffered in 1969. (Sources: Wikipedia, [4]).

1.5 Summary and comments

In the beginning of this chapter, in section 1.1, a definition of Progressive Collapse is given and elaborated on.
Then the characteristics that are desirable in a structure in order to reduce the chances of occurrence of Progressive Collapse, or to mitigate its intensity, are listed in section 1.2.
Then the causes of Collapses are listed, subdivided in categories, in section 1.3.
Finally, some of the most important case studies of Progressive Collapse are described and analyzed, as well as a case in which an initial Damage did not prompt a Collapse, in section 1.4.
All this information is useful to better understand the problem this work is about, as well as other parts of the work.

In the first section it is highlighted that the generally accepted definitions of Progressive Collapse include some ambiguities, and that the present work intends to overcome this problem by incorporating Progressive Collapse in a probabilistic Risk framework.

From the other sections it can be noticed that several types of Progressive Collapse exist, as well as different types of prompting events and types of structure behavior.

- The Ronan Point tower was a precast concrete panel building with a prevalent vertical conformation. Its Collapse prompting event was an accidental explosion due to a gas leak. Its main negative qualities were lack of ductility and redundancy; poor workmanship also contributed to its propensity to Collapse.
- The Hyatt Regency Collapse involved two walkways, whose structure consisted in steel frame and hangers. The prompting event of the Collapse was the live loads, which were actually well under the design values. The structure suffered lack of adequate design and poor workmanship. Furthermore, once the first connection failure occurred the non-redundant design made it impossible to stop the spreading of the Damage.
- The A.P. Murrah building had a cast in place concrete frame structure. Its Collapse was prompted by a terroristic bomb attack. It was made in accordance with the regulations, with good materials and workmanship; nevertheless, its inherent lack of redundancy and ductility made it extremely vulnerable to Progressive Collapse.
- The WTC1 and WTC2 towers were high rise steel frame buildings. Their Collapses were prompted by extensive impact Damage due to a terroristic attack and the subsequent extended fires. The structures were inherently very redundant and would have resisted the impact Damage without Collapse in absence of fire.
- The WTC7 building was a steel frame high rise building. Its Collapse was prompted by fire alone. Even tough it was designed in accordance with the building codes, the area in which the Collapse started was unable to resist the removal of few structural elements.
- The C.W. Post College dome (described in section 1.3) was a steel truss dome. Its Collapse was prompted by vertical loads lower than the design values. The model used in the design was not adequate to the actual structure.
- The Ambiance Plaza Collapse (figure 1.4) involved a multi-story building during construction. It was likely prompted by a lifting device failure. The adjacent devices were not able to withstand the increased loads and, once the Collapse started, the structure did not have enough resources to stop it.

In some other cases Progressive Collapse did not ensue.

- The Eads bridge is a steel truss structure purposely made very redundant. In two cases it was able to withstand localized Damage with no spreading or loss of functionality.
- The Empire State Building (described in section 1.4.4.1) is a steel frame high rise building which was hit by an aircraft with no subsequent Collapse. Its high level of redundancy is

deemed as the main cause of its Collapse resistance.
- The Confederation bridge (described in section 1.2) is a pre-stressed concrete, multi-span bridge that has been designed to stop Progressive Collapse trough compartmentalization, in case one of its piers gets damaged.

Historically, some of these cases prompted changes in building regulations and influenced research efforts, as chapter 2 illustrates.

Chapter 2 - Mitigation Strategies and Regulations

In this chapter, section 2.1 lists and explains the strategies that have been devised for Progressive Collapse mitigation.
Section 2.2 lists some of the most significant regulations about Progressive Collapse and explains how they evolved.
Section 2.3 summarizes and comments the chapter.

2.1 Mitigation strategies

Four categories of Progressive Collapse mitigation strategies have been identified. In general, they can be applied alone or combined together in the attempt to avoid the occurrence of Progressive Collapse or to mitigate its consequences. Each strategy has pros and cons, as well as limitations in their applicability, as explained in the next sections.

2.1.1 Event Control (EC)

This strategy consists in trying to foresee all the possible traumatic events that can cause an initial Damage and try to avoid their occurrence. Examples of this strategy are the standoff distance on strategic buildings, which prevents Damage from car bombs, or barriers that protect a structure from vehicular impact.

If successfully applied, this strategy should avoid any Damage, both direct and indirect. In reality, it is not always possible to foresee all the traumatic events that can happen, so it should be used in combination with other strategies. However, this strategy is often the simplest and most effective one against clearly identified potential traumatic events.
In some cases the causes of the traumatic events can be foreseen, but not avoided. For example, in [35] Ellingwood states that "*the writer is aware of one jurisdiction in which it was suggested that the use of natural gas be prohibited in certain multi-family residential buildings over four stories in height; the socioeconomic impact of that ordinance, had it been implemented, would have been unacceptable*". The reference is to the prompting event of the Ronan Point tower Collapse (section 1.4.1).

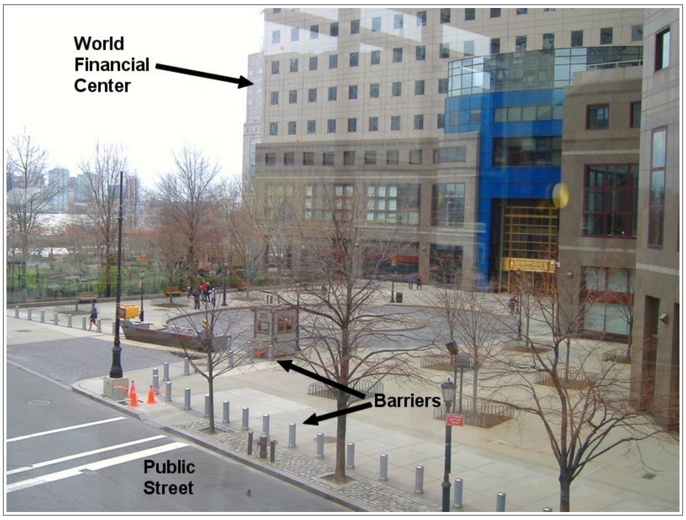

Figure 2.1: The barriers surrounding the World Financial Center in New York, NY, USA, prevent unauthorized vehicles to get close to the buildings and are a form of Event Control.

2.1.2 Specific Load Resistance (SLR)

This strategy consists in trying to foresee all the possible traumatic events as well as their intensities, and then design the structure to withstand these exceptional loads. An example of this is designing a structure to be able to withstand the overpressure of a bomb explosion or the force of a vehicular impact.

Like Event Control, the Specific Load Resistance strategy has the drawback that it is not always possible to foresee all the traumatic events. Furthermore, the intensities of the traumatic events could be underestimated, making the strategy ineffective.
One more limitation of this strategy is that the extra costs required to provide additional resistance can be deemed unacceptable, because the probability of occurrence of the exceptional events is generally low. It also must be noticed that designing a structure to resist a given type of event does not automatically make it safer against other types of events.
Thus, this strategy is best applied when some Hazards and their maximum intensities are clearly identified. For example, in a structure located close to the transit of vehicles of known characteristics (mass, maximum speed).

2.1.3 Alternate Load Path (ALP)

This strategy consists in modeling the structure with some structural elements removed to simulate the initial Damage, and verifying that the loads once borne by the damaged area can flow to the ground through an "alternate path" provided by the remaining elements. In some cases even elements that are commonly considered "non-structural", such as wall panels in a frame structure, are considered as possible load paths (figure 2.2). Furthermore, in an Alternate Load Path analysis the so called "catenary action" or "membrane action" of structural elements can be considered, as illustrated in figure 2.3.

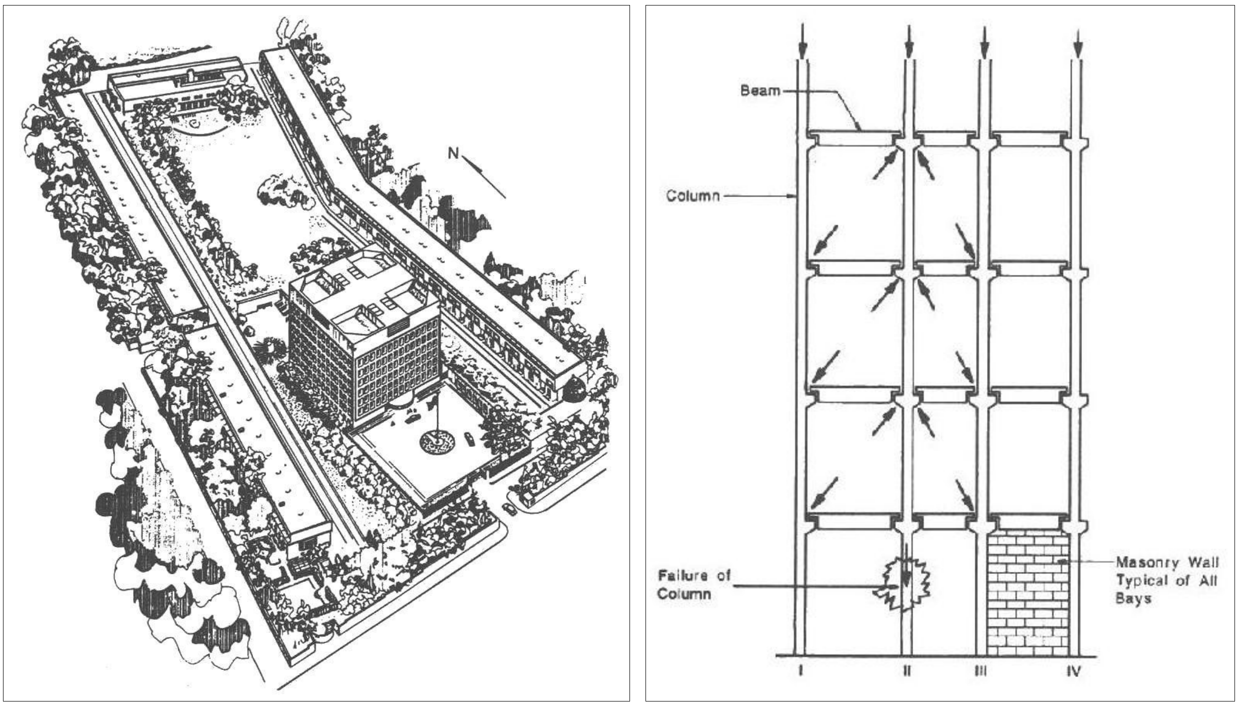

Figure 2.2: In 1986 the US Congress ordered a structural safety analysis of the building destined to be the US embassy in Moscow, Russia (then Soviet Union), shown at left. The analysis included a Progressive Collapse analysis, which was carried out following the ANSI A58 standard. Most beam connections of the structure had very little bending moment resistance, if any. Thus, the ALP method was applied by verifying that the loads of a removed column can flow to the surrounding columns through the masonry walls enclosed in the frame, acting as struts, as shown at right. (Sources: [53], [54]).

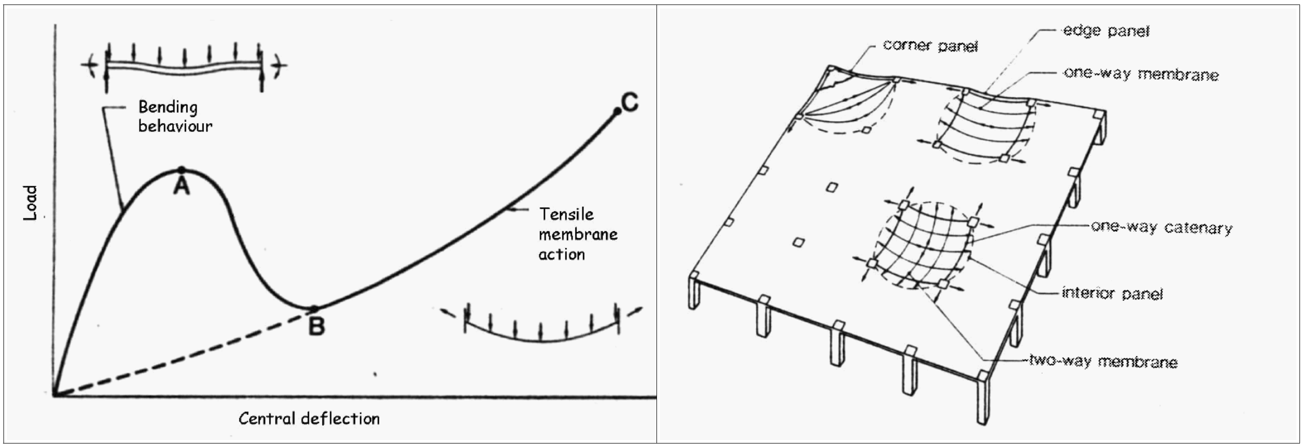

Figure 2.3: Representations of the so called "catenary action" or "membrane action". If a floor slab or a beam is overloaded, it can reach its ultimate limit state and lose most or all of its flexural resistance. In this case, the elements might still be able to support the loads by working in tension, similarly to a tensile structure. Reinforced concrete beams and floor slabs are a typical example of this: an overloading will generally cause crushing of compressed concrete and loss of bond between concrete and reinforcement, impairing the flexural resistance. At this stage, the metal reinforcement could still work in tension, if it is sufficiently continuous and restrained. (Source: [37]).

One advantage of this strategy is that by modeling the structure it can be obtained information about its behavior, for example highlighting critical areas. Another advantage is that in general the initial Damage is assumed without considering the event that caused it, thus guessing the traumatic events is not required.
One drawback of the Alternate Load Path strategy is that it generally requires more computational effort than the other ones. Furthermore, there is not general consensus on several aspects of its application, like for example: where and how extended the initial Damage should be; the required level of detail of the model; if and how the dynamic effects are to be modeled; what type of performance of the wounded structure is to be considered acceptable.

2.1.4 Indirect Design Method (IDM)

With this method, general structural integrity is pursued by prescribing measures such as minimum levels of strength, ductility and continuity.

This strategy is relatively simple to apply, as it does not require to foresee the traumatic events or to model the damaged structure. It is generally deemed appropriate for regular building layouts and lower importance structures. On the other side, since the structure is not analyzed there is no guarantee that the measures will actually be effective.

2.2 Regulations about Progressive Collapse

This section reports an historical overview of the regulations about Progressive Collapse and describes the current status. Two important research works are also related in sub-section 2.2.1.1. Most of the historical information is taken from [24] and [35].

2.2.1 Historical overview

The first reference to Progressive Collapse in a building regulation was probably the April 1967 Comité Européen du Béton (CEB) "*Recommandations Internationales Unifiées Pour le Calcul et*

l'Exécution des Constructions en Panneaux Assembles de Grand Format" ("*Unified International Recommendations for the Design and Construction of Large-Panel Structures*"), which included the following text (translated from French):
"*One can hardly over-emphasize the absolute necessity of effectively joining the various components of the structure together in order to obviate any possible tendency for it to behave like a 'house of cards' and of organizing the structure accordingly. In this respect it would appear to be of major importance to install mechanically continuous steel ties interconnecting opposite walls or facades and providing safeguards for all the vertical panels*."

Only after the Ronan Point Collapse (May 1968; see section 1.4.1 for its description and analysis) widespread attention was gained by the "house of cards" phenomenon, which was termed "Progressive Collapse".
At the time in the United Kingdom no regulation considered it. The board of inquiry report (Griffiths, Pugsley, Saunders - *Report of the inquiry into the collapse of flats at Ronan Point, Canning Town* - Her Majesty's Stationary Office, London, UK 1968) found that there was no violation of any applicable building standard in the design of the Tower, yet it was not "*an acceptable building*". The report highlighted that the building standards, typically, gave detailed requirements for the design of individual members but provided little guidance for the stability design of the entire structural system.

Subsequently, the circular 62/68 entitled "*Flats Constructed with Precast Concrete Panels. Appraisal and Strengthening of Existing High Blocks; Design of New Blocks*", issued on November 15, 1968 by the UK Ministry of Housing and Local Government was the first document to include recommendations on the prevention of similar events. The circular required that multistory buildings be designed to provide either an alternate load path in the event of the loss of a single critical member or sufficient local resistance to withstand the effects of a 5 psi (34 kPa) design pressure. These two requirements correspond to the Alternate Load Path (ALP) and Specific Load Resistance (SLR) mitigation strategies, which are described in sections 2.1.3 and 2.1.2 of the present text, respectively. The value 5 psi was based on a gas-type explosion, although this is not stated in the document.

The requirements of circular 62/68 were then adopted in the "*Statutory Instrument 1970 No. 109 - The Building (Fifth Amendment) Regulations*" issued by Her Majesty's Stationery Office in 1970. The "Fifth Amendment" immediately received considerable criticism, both inside and outside the UK. The local resistance approach was was criticized on the basis of of the lack of information about the the dynamic effect of an abnormal load; due to this insufficient knowledge, the specified design static pressure 5 psi was considered unjustified. It was argued that the required overpressure would not necessarily protect against other abnormal events such as vehicular collisions. The alternate load path approach was deemed too complex, as well as illogical, since more than one critical load-carrying member could be removed by an abnormal event. The provisions were also criticized for being implemented without knowing the effect on the cost of building construction.

Despite these objections, the Ronan Point Collapse affected the building regulations of many other countries.
In 1970, Canada adopted a Progressive Collapse requirement in its National Building Code.
In the USA, in 1972 the ANSI A58.1 standard "*Minimum Design Loads in Buildings and Other Structures*" included the following general recommendation:
"*Buildings and structural systems shall provide such structural integrity that the hazards associated with progressive collapse such as that due to local failure caused by severe overloads or abnormal loads not specifically covered herein are reduced to a level consistent with good engineering*

practice."
In 1973 the City of New York amended its building code to require that Progressive Collapse resistance be provided by either the Specific Load Resistance or the Alternate Load Path methods. Provisions for structural ties entered the British Standards in 1974.

In the USA a research program denominated "Operation Breakthrough" was launched on May 8, 1969, by the Department of Housing and Urban Development (HUD) to encourage industrialized housing concepts. Progressive Collapse was considered a concern for the evaluation of concrete panel systems, and the criteria adopted by HUD were similar to the British Fifth Amendment: "*Explosions or other catastrophic loads on any one story level should not cause progressive structural collapse at other levels. The criterion applies to buildings four stories or higher. At a load level of 1.0 dead + 0.5 live, the accidental removal of any one of the following (load) supporting structural elements at one level should not cause collapse of the structure on another level:*

a) two adjacent wall panels forming an exterior corner;
b) one wall panel in a location other than an exterior corner;
c) one column or other element of the primary structural support system.

This criterion is waived if the above-mentioned structural element or elements are capable of resisting a pressure of 5 psi (34.5 kPa), applied in the most critical manner within one story level to one face of the element and of all space dividers supported by the element or attached to it."
(From "*Guide Criteria for the Evaluation of Operation Breakthrough Housing System*" Building Research Division Team, Springfield, VA, 1970).

The 1971 HUD-FHA (Federal Housing Authority) "*Provisions to Prevent Progressive Collapse*" included other "Breakthrough" criteria:
"*Joints between prefabricated structural elements used as columns, beams, bearing walls, or slabs should develop continuity similar to that provided by conventional cast-in-place concrete or structural steel framing systems. In regions not subject to severe seismic or wind action, connections should not be designed solely as gravity-type relying only on compression and friction.*"
Furthermore, the 1971 HUD-FHA criteria stated that, if abnormal loading occurred, Damage must be limited to 93 m^2 (1000 ft^2) or 20% of horizontal floor area, whichever was less, and to three stories vertically.

2.2.1.1 Two influential research works

It is worth to mention two works that influenced building regulations and research.

The first work is "*Progressive Collapse, Abnormal Loads and Building Codes*", a paper by D.E. Allen and W.R. Schriever included in the book "*Structural Failures: Modes, Causes, Responsibilities*" [4], published in 1973.
The paper estimates the incidence of Progressive Collapses from two news sources: the Engineering News Record (from 1968 to 1972) and from newspapers clippings on Collapses in Canada (from 1962 to 1972). The "rule of three" is used in classifying structural Collapses as progressive or not: a Collapse is considered progressive if it involves members that are three or more members away from the original failure or if three or more spans collapse.
According to the these statistics (as reported in table 2.1), the number of Progressive Collapses comprises approximately 15% (i.e., 75 over 495) to 20% (22 over 110) of the total number of Collapses. This statistic on the incidence of Progressive Collapses is still quoted today in regulations and research works, like for example in the standard ASCE 7-05 (which is described in

section 2.2.2.2 of the present work and is a reference for other regulations).

TABLE 1.—NEWS INCIDENTS INVOLVING PROGRESSIVE COLLAPSE

Collapse Designation (1)	*Engineering News Record* (4 years) (2)	Canada (10 years) (3)
During Construction		
Due to impact, explosion	2	1
Formwork, bracing or erection error	10	35
Design error	1	0
During Service Life		
Due to explosion	1	0
Due to impact	4	8
Design, manufacture or construction error	3	22
During Demolition, Adjacent Excavation	1	6
TOTAL	22	75
Total News Incidents Involving All Types of Collapse	110	495

Table 2.1: Summary of the statistics collected by Allen and Schriever. (Source: [4]).

The second work is "*The Incidence of Abnormal Loading in Residential Buildings*" by E.V. Leyendecker and E.F.P. Burnett [23], published in 1976.
The work presents the findings of an analysis of available US statistics concerning the incidence of abnormal loading events in residential buildings. The study evaluates natural gas explosions, bomb explosions, motor vehicle collisions, sonic boom (i.e., shock waves generated by aircrafts flying at supersonic speeds), aircraft collision and explosion of hazardous materials. It is concluded that only the gas-related explosions, bomb explosions and vehicular collision are of significance in building design for Progressive Collapse. For the other Hazards, it is suggested that certain building locations may require a site study. The work also states that the considered abnormal loads do not constitute a complete list, and other types may occur.
The following excerpt explains how the data were collected:
"The number of incidents causing damage in excess of a specific level (measured in dollars) or a brief description of damage is usually provided. There is rarely complete information on the type or number of structures affected or an adequate description of their damage. There is also no available US data which provide a correlation between a specified damage level and a corresponding damage loss in dollars. Finally there are no data on the load characteristics of the actual events.
In order to compare the data for the gas explosion, bomb explosion, and vehicular collision, the following definitions are adopted:
1) *Total incidents- All incidents involving a particular abnormal load.*
2) *Intermediate or greater damage – Damage in excess of $1000 for gas explosions, $1000 for vehicular collision, or described as intermediate or greater for bomb explosions. This level implies fairly extensive damage such as walls blown down.*
3) *Severe damage – Damage in excess of $10000 for gas explosions; $5000 for vehicular collision; or describes as severe for bomb explosions. This level implies extensive structural damage, such as unit destroyed.*

These definitions involve some arbitrary judgment because of the limitations on the available data".

The collected data are discussed in terms of probability theory. The following equation is given:

$$P(D)_p = P(D_p|AB) \cdot P(AB) \qquad (2.1)$$

where $P(D)_p$ is the probability of Damage above a specified level;
the subscript p indicates the specified Damage level (intermediate or severe);
$P(AB)$ is the probability of occurrence of an abnormal load event;
$P(D_p|AB)$ is the probability of Damage above a specified level given an abnormal load event.

The terms $P(D)_p$ and $P(AB)$ are obtained from the data reported in figure 2.4, while the other term is computed as

$$P(D_p|AB) = P(D)_p / P(AB) \qquad (2.2)$$

The calculated probabilities are reported in table 2.2.

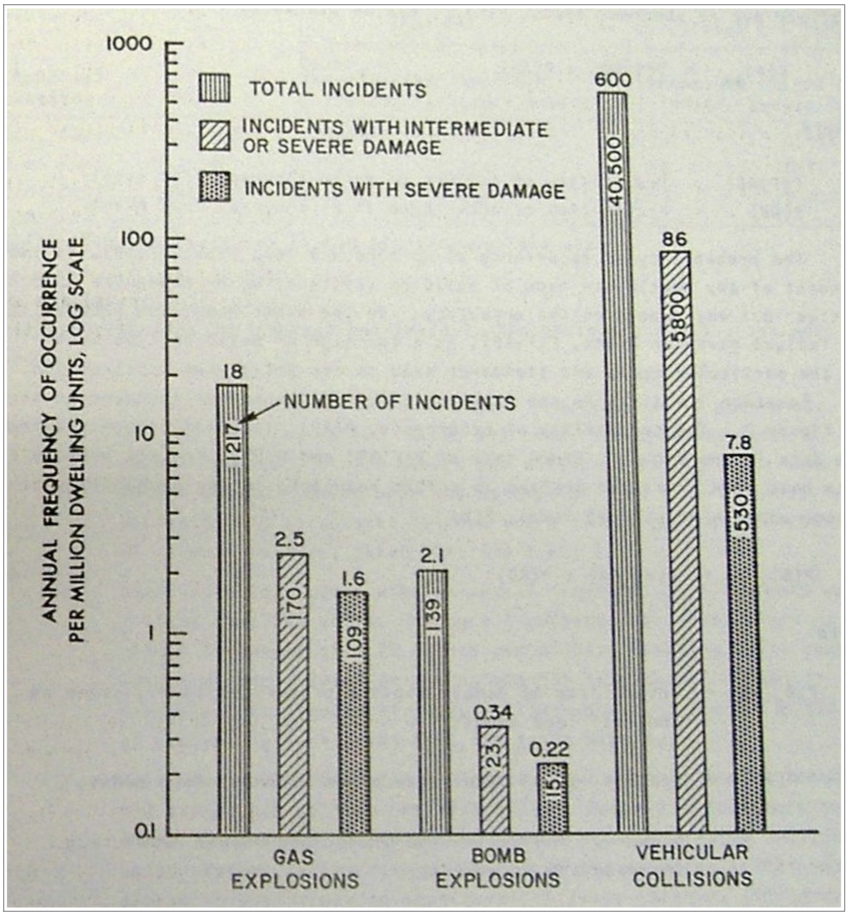

Figure 2.4: Summary of annual probabilities of abnormal loadings for 1970. (Source: [24]).

Abnormal Loading	P(AB)	$P(D)_p$		Calculated $P(D_p \mid AB)$	
		Based on Intermediate Damage	Based on Severe Damage	Based on Intermediate Damage	Based On Severe Damage
Gas. Expl.	18×10^{-6}	2.5×10^{-6}	1.6×10^{-6}	0.14	0.089
Bomb Expl.	2.1×10^{-6}	0.34×10^{-6}	0.22×10^{-6}	0.16	0.11
Veh. Coll.	600×10^{-6}	86.0×10^{-6}	7.8×10^{-6}	0.14	0.013

NOTE: The probabilities listed are per dwelling unit.

Table 2.2: Calculated estimates of the probabilities in Leyendecker and Burnett's study. (Source: [24]).

The work also states that "*a number of assumptions have been necessary in order to analyze the data presented in this report. These are related primarily to the lack of detailed descriptions of damage accompanying the various abnormal loading events and the lack of detailed descriptions of the buildings involved in the incidents*" and "*the statistical reporting of abnormal loading events needs to be considerably improved in order to obtain load data and damage data in particular types of building construction*". In spite of this, not much has been done, and Leyendecker and Burnett's statistics are still referenced today, for example in [35].

2.2.2 Current status

Nowadays, the regulations of many countries include recommendations or provisions about Progressive Collapse. The following is a partial list:

- The British provisions, with modifications that put less emphasis on explosions and more on ductile performance, are still in force today in the UK. Namely, the notional removal of an essential structural element should cause only local Collapse ($70m^2$ or $750ft^2$ or 15% of the plan area of the story, whichever is less), and buildings should be designed for an accidental pressure of 34kPa or 5psi acting simultaneously with dead and imposed loads.
- The current Italian regulations give a definition of Robustness as a general requirement, which is considered guaranteed by respecting the prescribed limit states and by applying conventional horizontal loads. The Italian regulations are further described in section 2.2.2.1.
- The National Building Code of Canada contains a general statement about the need for structural integrity, and its Commentary provides an extensive discussion on means to achieve that goal. The extent of the discussion reflects the importance accorded to the topic at the time, e.g., the 1975 version is much longer than the 1995 version. The Commentary covers recommendation for good structural layout, continuity of reinforcement, and structural mechanisms that would mitigate Progressive Collapse after local loss of support. No specific values are given for tie forces or accidental loads for key structural elements.
- The ACI 31-05 standard is an example of indirect design. It defines requirements for structural integrity such as continuity of reinforcement and use of ties in precast concrete construction.
- On the other side, the 1998 New York City Building Code is an example of direct design. It only mentions the alternate load path and the specific local resistance (34 kPa or 5 psi) methods.
- The commentary of ASCE 7-05 contains extensive discussion on General Structural Integrity. It lists the direct design approaches (Alternate Path and Specific Load Resistance methods) and the Indirect Design approach. It provides design guidelines for general structural integrity, such as good plan layout and use of structural ties. As well, it recommends load combinations including extraordinary loads, and explains the underlying probabilities. The ASCE 7-05

standard is further described in section 2.2.2.2

Furthermore, two regulations do exist, which include precise procedures to determine if a structure is to be deemed acceptable with respect to Progressive Collapse. They are the Department of Defense's (DOD) Unified Facilities Criteria (UFC) "*Design of Buildings to Resist Progressive Collapse*" - DoD UFC 4-023-03 [12], [13], and the General Services Administration's guidelines "*Progressive Collapse Analysis and Design Guidelines for New Federal Office Buildings and Major Modernization Projects*" (GSA 2003) [15]. Their enforcement is only mandatory for buildings that belong to these two agencies. However, since no other regulation gives such detailed procedures, they are also used outside. The 2005 and 2009 versions of DoD UFC 4-023-03 are described in sections 2.2.2.3 and 2.2.2.4, respectively. The GSA 2003 guidelines are described in section 2.2.2.5.

2.2.2.1 Italy

In Italy, before 2005 the regulation on precast constructions (DM 03.10.1987) requested that "*the designer shall give particular attention in order to contain the propagation of a local impairment (chain-like collapse)*". The regulation also required minimum ties and generally requested that, in case of exceptional events "*the prospective destruction of a vertical bearing element of the size of a room, of two such elements in a corner position, shall not allow a chain-like collapse*".

In 2005, all the Italian building regulations were replaced by a single Text ("*Norme Tecniche per le Costruzioni*", commonly referred as "*Testo Unitario*"), which included multiple references to structural Robustness. A second version of the Text, which replaced the first one, was released in 2008 and included much fewer references and requirements.

The 2005 version of the Text included two definitions of the term "Robustness" ("*the ability to avoid damages non proportional to the entity of the prompting causes, such as fire, explosions, impacts or consequences of human errors*", given in section 2.1, and "*the ability of the structure to respond in a proportional manner to exceptional situations, whose occurrence cannot be excluded, but that cannot be entirely described*", given in section 3.1.

Section 6.1 requested that "*buildings must be designed so that the principal structural system can withstand local damage without suffering total collapse; the decay in the resistance performance must be proportional to the cause that provoked it*" saying that this can be achieved through "*a layout of the structural elements that maintains resistance and stability of the principal scheme through a transfer of the load from any damaged structural region to the adjacent ones; this can be achieved by providing sufficient continuity, static indeterminacy, ductility to the parts of the building*."

Furthermore, "*this way, it should also be prevented the diffusion of the damage from a limited region of the structure to a significant part of it, or even to the entire structural organism, in the so called "progressive collapse" modality. Such collapse modality, and in general the damage propagation, will also be achieved through an appropriate compartmentalization of the structural organism.*" It must be noticed that the previous quoted part includes the expressions "*progressive collapse*" and "*compartmentalization*", but the text does not give a precise definition of "*progressive collapse*", and "*compartmentalization*" is only defined referring to thermal insulation of structural elements.

The same section says that the Robustness of a work must be tested by imposing some nominal loads, "*arbitrary but significant for the considered scenario*", including an horizontal load which is generally assumed as a fraction of the vertical loads, and by imposing "*lack of structural elements, to evaluate the consequence of their loss regardless of the cause, in order to locate the critical ones*". It must be noticed that these last two prescriptions should have been both mandatory (i.e., the Robustness of a work *must* be tested, by imposing some nominal loads *and* by imposing lack of structural elements). Furthermore, the last prescription corresponds to the Alternate Load Path mitigation strategy (described in section 2.1.4 of the present work), but no indication is given for load combinations to be applied in the test, and to the number of elements to be removed from the model. This was an exception, as most regulations prescribe combinations and the removal of a single element at a time.

The 2008 version of the Text includes one definition of Robustness, almost equal to the first one of the 2005 Text: "*the ability to avoid damages non proportional to the entity of the prompting causes, such as fire, explosions, impacts.*"
The requirement of Robustness is considered guaranteed by respecting the limit states prescribed by the regulation and by applying conventional horizontal loads equal to 1% of the other loads, not concurrently with wind and earthquake. A similar prescription was given in the 2005 Text, but the entity of the horizontal loads was reduced for structures taller than 100 m.
Furthermore, "*for exceptional design scenarios, the design shall demonstrate the robustness of the work by means of procedures of damage scenarios*" with given safety coefficients of the materials. Fires, explosions and impacts are classified as exceptional loads. Load combinations that include exceptional loads are given. When explicit consideration is not given to exceptional loads, "*the structural layout, the details and the used materials will have to be such to avoid that the structure could be damaged disproportionately with respect to the cause*".
In the section of the Text is about design criteria for explosions, it says that "*local damages due to explosions, even severe, are considered acceptable if they don't put into danger the entire structure or if structural integrity is maintained long enough to put into effect the necessary emergency measures*". In the same section, compartmentalization by disconnection is described among other protection measures (see section 1.2 of the present work for a description of this technique).

2.2.2.2 ASCE 7

Nowadays one of the most considered regulations is the ASCE Standard 7 "*Minimum Design Loads for Buildings and Other Structures*" (formerly ANSI Standard A58). It includes a small section about "*General Structural Integrity*", as well as a commentary that elaborates extensively. Among the other things, the commentary lists the mitigation strategies, provides design guidelines and recommends load combinations for the Specific Load Resistance and Alternate Load Path methods. Applying the guidelines and recommendations of ASCE 7 is not mandatory, as they are included in a commentary; however, some of these have been included in other regulations, such as the DoD UFC 4-023-03 and GSA 2003 (which are described in the next three sections of this work). Furthermore, the Standard does not provide specific design criteria (such as e.g. the intensity of the extraordinary loads or the required tie strengths).

The edition of the Standard that first introduced a requirement for Progressive Collapse due to "*local failure caused by severe overloads*" was the American National Standards Institute (ANSI) A58.1-1972 Standard, which was the first following the Ronan Point Collapse. Additional commentary was provided in later editions. A requirement to check strength and stability of structural systems under low-probability events was introduced in 1995.

The 1998 edition of the American Society of Civil Engineers Standard 7 (ASCE 7-98) included a short discussion on mitigation of Progressive Collapse in its commentary section C1.4. In particular, it recommended:
- to provide sufficient continuity, redundancy, or energy dissipating capacity (ductility), or a combination thereof, in the members of the structure;
- to identify extraordinary events with a probability of occurrence in the range of 10^{-6}/yr to 10^{-4}/yr or greater, and ensure key load-bearing elements can withstand such events;

– that minimum tie force between structural elements should be 20 kN/m (1400 lb/ft);
– that "*design limit states include loss of equilibrium as a rigid body, large deformations leading to significant second order effects, yielding or rupture of members, of connections, formation of a mechanism, instability of members or the structure as a whole. […] In a damaged structure, additional load-carrying mechanisms, such as membrane or catenary action, may be included*";
– that, as elastic analysis may vastly underestimate the capacity of the structure, non-linear or plastic analysis may be used.

The 2002 and 2005 editions (ASCE 7-02 and ASCE 7-05) are almost identical, with only small editorial differences. Compared to the 1998 version, the commentary is expanded considerably and provides guidance on various structural actions to prevent local Damage from progressing. Interestingly, the commentary no longer provides guidance on the minimum tie force.

Section C1.4 of the commentary ("*General Structural Integrity*") lists the Alternate Path and the Specific Local Resistance methods (which together form the "Direct Design" category) as well as the Indirect Design method, as "*ways to obtain resistance to progressive collapse*". It is also comments that "*alternate path studies may be used as guides for developing rules for the minimum levels of continuity and ductility needed to apply the indirect design approach to enhance general structural integrity*".
The Ronan Point and A.P. Murrah Collapses are also briefly described, as "Examples *of General Collapse*".
Guidelines for the provision of general structural integrity are then given. These include:
1. Good plan layout.
2. Provide an integrated system of ties among the principal elements of the structural system.
3. Returns on walls. A return is a short length of wall usually at right angle to another wall.
4. Changing directions of span of floor slab.
5. Load-bearing interior partitions.
6. Catenary action of floor slab.
7. Beam action of walls. Walls may be assumed to act as the web of a beam with the slabs above and below acting as flanges.
8. Redundant structural systems.
9. Ductile detailing.
10. Provide additional reinforcement to resist blast and load reversal when blast loads are considered in design.
11. Consider the use of compartmentalized construction in combination with special moment-resisting frames in the design of new buildings when considering blast protection.

Section C2.5 of the commentary ("*Load Combinations for Extraordinary Events*") recommends checking the capacity of the structure after notional removal of load-bearing elements (i.e., using the Alternate Load Path strategy) with the following load combination:

$$(0.9 \text{ or } 1.2)\ D + (0.5\ L \text{ or } 0.2\ S) + 0.2\ W \tag{2.3}$$

where D is the dead load, L is the live load, S is the snow load and W is the wind load. According to Ellingwood [35], this load combination is obtained from load statistics and principles of reliability analysis, and it has an annual probability of being exceeded of approximately 0.05.

For checking the capacity of a structure or structural element to withstand the effect of an extraordinary event (Specific Load Resistance strategy), the following load combinations are recommended:

$$1.2\ D + A_k + (0.5\ L \text{ or } 0.2\ S) \qquad (2.4)$$
$$(0.9 \text{ or } 1.2)\ D + A_k + 0.2\ W \qquad (2.5)$$

where D is the dead load, L is the live load, S is the snow load, W is the wind load and A_k is the value of the load or load effect resulting from extraordinary event A. The value of A_k "*must be specified by the authority having jurisdiction*".

Further considerations given by the commentary are the following:
"*Generally, extraordinary events with a probability of occurrence in the range 10^{-6} to 10^{-4}/yr or greater should be identified, and measures should be taken to ensure that the performance of key load-bearing structural systems and components is sufficient to withstand such events.*"
"*Specific design provisions to control the effect of extraordinary loads and risk of progressive failure can be developed with a probabilistic basis. One can either attempt to reduce the likelihood of the extraordinary event or design the structure to withstand or absorb damage from the event if it occurs. Let F be the event of failure and A be the event that a structurally damaging event occurs. The probability of failure due to event A is*

$$P = P(F \mid A)\, P(A) \qquad (C2.5\text{-}1)$$

in which P(F | A) is the conditional probability of failure of a damaged structure and P(A) is the probability of occurrence of event A. The separation of P(F | A) and P(A) allows one to focus on strategies for reducing risk. P(A) depends on siting, controlling the use of hazardous substances, limiting access, and other actions that are essentially independent of structural design. In contrast, P(F | A) depends on structural design measures ranging from minimum provisions for continuity to a complete post-damage structural evaluation."
"*The probability, P(A), depends on the specific hazard. Limited data for severe fires, gas explosions, bomb explosions, and vehicular collisions indicate that the event probability depends on building size, measured in dwelling units or square footage, and ranges from about 0.23×10^{-6} / dwelling unit/year to about 7.8×10^{-6} /dwelling unit/year.*"
"*If one were to set the conditional limit state probability P(F | A) = 0.1/yr - 0.2/yr, however, the annual probability of structural failure from eq. C2.5-1 would be on the order of 10^{-7} to 10^{-6}, placing the risk in the low-magnitude background along with risks from rare accidents.*"
"*Design requirements corresponding to this desired P(F | A) = 0.1 – 0.2 can be developed using first-order reliability analysis if the limit state function describing structural behavior is available.*"

2.2.2.3 DoD 2005

The 2005 version of the Department of Defense's (DOD) Unified Facilities Criteria (UFC) "*Design of Buildings to Resist Progressive Collapse*" - DoD UFC 4-023-03 (in the following referred as DoD 2005 for brevity) is described in this section. In 2009 it has been replaced by a new version, which is described in section 2.2.2.4 of the present work, and which includes several important changes.

DoD 2005 requires that "*all new and existing buildings of three stories or more be designed to avoid progressive collapse*."
The Progressive Collapse design requirements use two approaches: tie forces (i.e., Indirect Design Method) and Alternate Load Path (the latter referred as "Alternate Path method", or AP). Specific Load Resistance and Event Control are not used.

Every DoD facility is assigned one of four Level of Protection ratings: Very Low Level Of Protection (VLLOP); Low Level Of Protection (LLOP); Medium Level Of Protection (MLOP) and

High Level Of Protection (HLOP). The level of Progressive Collapse design for a structure is correlated to its Level of Protection.

The Alternate Path method is used in two situations:

- when a vertical structural element cannot provide the required tie strength, the AP method can be used to determine if the structure can bridge over the deficient element after it has been notionally removed;
- for structures that require Medium (MLOP) or High Levels of Protection (HLOP) the Alternate Path method must be applied for the removal of specific vertical load-bearing elements. A peer review for all these Alternate Path analyses must be performed and documented by independent and qualified organizations.

More specifically:

- A structure with a Very Low Level of Protection (VLLOP) must provide a specified adequate horizontal tie force capacity. If a structural element does not provide the required horizontal tie force capacity, it must be re-designed in the case of new construction or retrofitted in the case of existing construction; the Alternate Path method cannot be used.
- A structure with a Low Level of Protection (LLOP) must incorporate both horizontal and vertical tie force capacities. If a vertical structural member cannot provide the required vertical tie force capacity, the designer must either re-design the member or use the Alternate Path method to prove that the structure can bridge over the element when it is removed. For elements with inadequate horizontal tie force capacity, the Alternate Path method cannot be used; in this case, the designer must redesign the element for new constructions or retrofit the element for existing constructions.
- For Medium (MLOP) and High Levels of Protection (HLOP) structures the requirements are similar to LLOP, plus the following: "*the structure must be able to bridge over specific vertical load-bearing elements that are notionally removed from the structure. The plan locations of the removed vertical load-bearing elements include, as a minimum, the center of the short side, the center of the long side, and the building corner. In addition, vertical loadbearing elements are removed wherever there is a significant variation or discontinuity in the structural geometry, such as re-entrant corners and abrupt changes in bay sizes.*"

  "For each plan location of a removed element, an Alternate Path analysis is performed for every floor, one at a time; thus, if there are three plan locations and eight stories, twenty four AP analyses must be performed. If bridging cannot be demonstrated for one of the removed load-bearing elements, the structure must be re-designed or retrofitted to increase the bridging capacity. Note that the structural re-design or retrofit is not applied to just the deficient element, i.e., if a structure cannot be shown to bridge over a removed typical column at the center of the long side, the engineer must develop suitable or similar re-designs or retrofits for that column and other similar columns. For instance, a re-design might consist of additional positive moment rebar at a reinforced concrete beam-column joint; this new design must be applied to other columns on that external column line."

  However, *"if the designer can show that similar structural response is expected for column removal on multiple floors (say, floors 4 though 10), the analysis for these floors can be omitted but the designer must document the justification for not performing these analyses."*

  Furthermore, "*for structures with underground parking or other uncontrolled public ground floor areas, remove internal columns near the middle of the short side, near the middle of the long side and at the corner of the uncontrolled space. [...] For each plan location, the AP analysis is only performed for the column on the ground floor or parking area floor and not for all stories in the structure*."

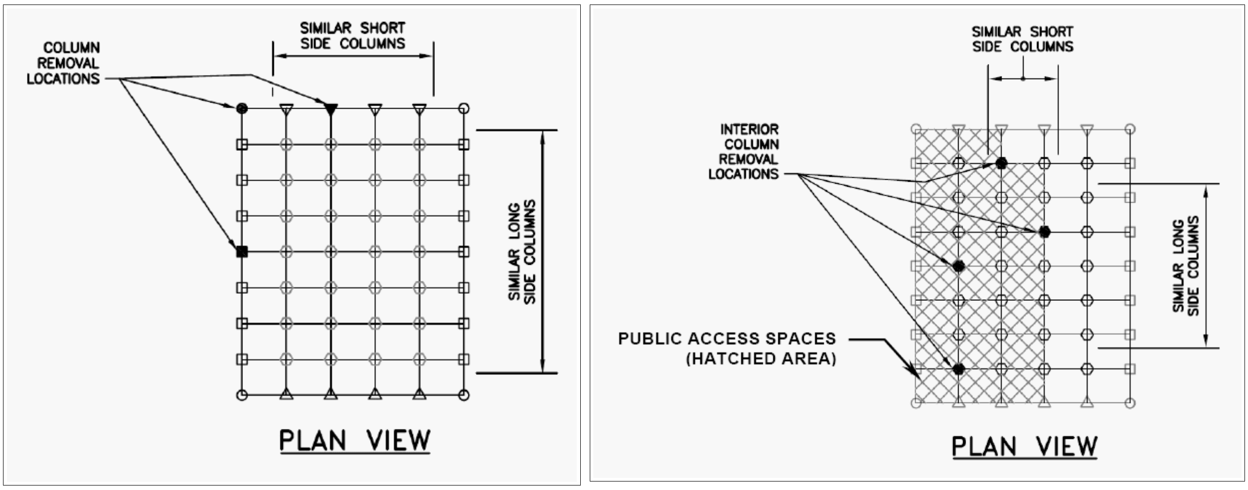

Figure 2.5: Location of external and internal columns to notionally remove in the Alternate Load Path analysis, according to DoD UFC 4-023-03. Similar schematics are given for load-bearing wall structures. (Source: [12]).

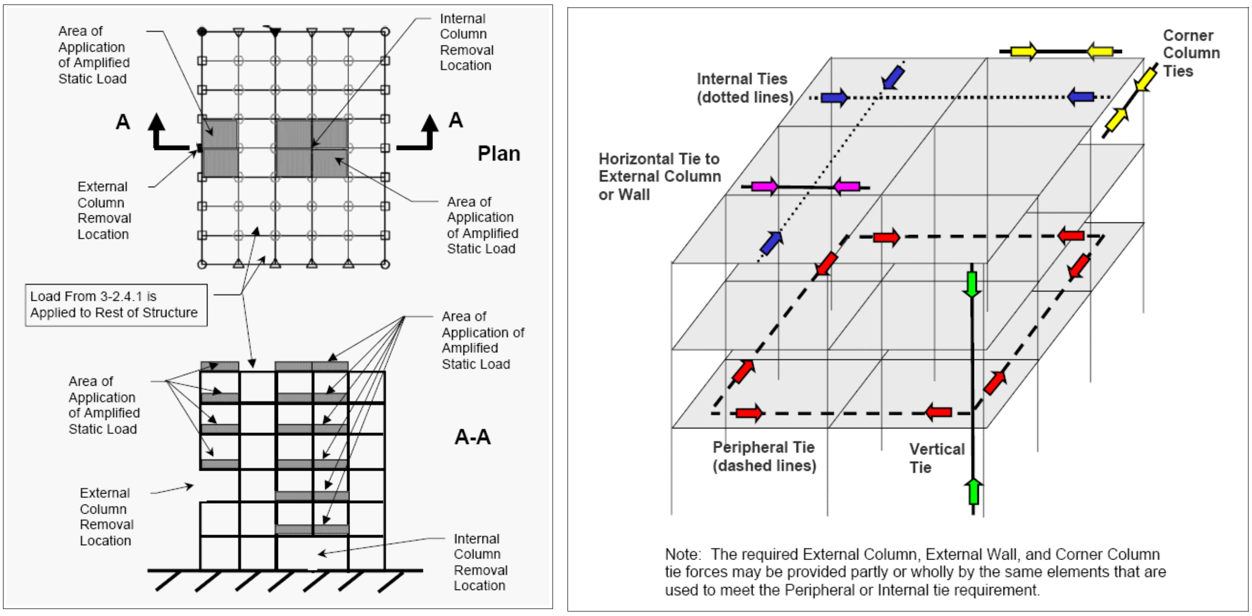

Figure 2.6: (Left) Location of the areas in which amplified static loads must be applied in an Alternate Path analysis, according to DoD UFC 4-023-03. (Right) Schematic of the ties required by the 2005 version of DoD UFC 4-023-03 (Source: [12]).

Specific sections contain the tie requirements as well as the acceptance criteria for Alternate Path analyses for reinforced concrete, structural steel, masonry, wood and cold-formed steel. The acceptance criteria include strength limits, deformation limits and spread of Damage limits. Step-by-step procedures for Linear Static, Nonlinear Static and Nonlinear Dynamic Analysis are given.

In section 3.2.4.2 of DoD 2005 the load combination to use for nonlinear dynamic Alternate Load Path is given:

$$(0.9 \text{ or } 1.2)\, D + (0.5\, L \text{ or } 0.2\, S) + 0.2\, W \tag{2.6}$$

where D is the dead load; L is the live load; S is the snow load and W is the wind load.
For linear and nonlinear static analyses, the following amplified load combination must be applied to the bays immediately adjacent to the removed element and at all floors above the removed element:

$$2.0\ [(0.9 \text{ or } 1.2)\ D + (0.5\ L \text{ or } 0.2\ S)] + 0.2\ W \tag{2.7}$$

for the rest of the structure, the load combination (2.6) must be used.
It must be noticed that the load combination (2.6) corresponds to the one given by ASCE 7, which is reported as formula (2.3) in the present work.

It is prescribed that, if an element reaches the strength or deformation limits during an Alternate Path analysis, then it must be either modified or removed.
"For Linear Static models, structural elements that can sustain a constant moment while undergoing continued deformation must be modified through insertion of an effective plastic hinge. Place a discrete hinge in the model at the location of yielding and apply two constant moments, one at each side of the discrete hinge, in the appropriate direction for the acting moment. Determine the location of the effective plastic hinge through engineering analysis and judgment or with the guidance provided for the particular construction type. In Nonlinear Static and Dynamic models, the software must have the ability to adequately represent the nonlinear flexural response, after the internal moment reaches the flexural design strength of the element." If the element must be removed, then "*Redistribute the loads associated with the failed element per Section 3-2.4.3, before the analysis continues."*
"3-2.4.3 Loads Associated with Failed Elements. *As discussed later, the internal forces or deformation in a structural element or connection may be shown to exceed the acceptability criteria. If so, the element is considered to be failed and is removed from the model."*
"For a Nonlinear Dynamic analysis, double the loads from the failed element to account for impact and apply them instantaneously to the section of the structure directly below the failed element, before the analysis continues. Apply the loads from the area supported by the failed element to an area equal to or smaller than the area from which they originated."
"For a Linear or Nonlinear Static analysis, if the loads on the failed element are already doubled as shown in Section 3-2.4.2, then the loads from the failed element are applied to the section of the structure directly below the failed element, before the analysis is re-run or continued. If the loads on the failed element are not doubled, then double them and apply them to the section of the structure directly below the failed element, before the analysis is re-run or continued. In both cases, apply the loads from the area supported by the failed element to an area equal to or smaller than the area from which they originated."

The following excerpts describe the prescribed Damage limits of the Alternate Load Path analyses.
"3-2.6.1 Damage Limits for Removal of External Column or Load- Bearing Wall
For the removal of a wall or column on the external envelope of a building, the Damage Limits require that the collapsed area of the floor directly above the removed element must be less than the smaller of 70m^2 (750ft^2) or 15 % of the total area of that floor and the floor directly beneath the removed element should not fail. In addition, any collapse must not extend beyond the structure tributary to the removed element."
"3-2.6.2 Damage Limits for Removal of Internal Column or Load- Bearing Wall
For the removal of an internal wall or column of a building, the Damage Limits require that the collapsed area of the floor directly above the removed element must be less than the smaller of 140m^2 (1500ft^2) or 30 % of the total area of that floor, and the floor directly beneath the removed element should not fail. In addition, any collapse must not extend beyond the bays immediately adjacent to the removed element."

In the commentary of DoD 2005, several observations are given.

Section B-3 states that the tie force requirements are very similar to those provided in the British Building Standards, which were developed in response to the Ronan Point accident in 1968. Furthermore, "*attempts to uncover the processes and logic by which these requirements were developed were partially successful and, in discussions with British engineers, it has been noted that engineering judgment was used for some of the requirements*." Results of the background research are presented.

Section B-4.1 is about the removal criteria of load-bearing elements.
"*As discussed in the UFC, the AP method for MLOP and HLOP requires that load-bearing elements be removed from every floor, after their plan location is identified. The main motivation for this requirement is that DoD facilities could be attacked with artillery, rockets, mortars, or rocket propelled grenades, all of which could damage a structure at upper floors. Many buildings are more susceptible to progressive collapse if the damage initiates at higher elevations (due to the reduced reserve capacity from the fewer number of floors above) and this requirement will motivate the designer to distribute additional strength and ductility to the upper levels.*"
Furthermore, "*the column or wall is removed from the structural model without degrading the capabilities of the joint at the upper end of the member. Physically, this is unlikely to happen in an accidental or man-made event and critics of this approach usually refer to the column deletion as the "immaculate removal." However, it should be emphasized that the AP method is not intended to replicate an actual event; the goal is to verify that the structure has satisfactory flexural resistance to allow bridging across an area with localized damage*".

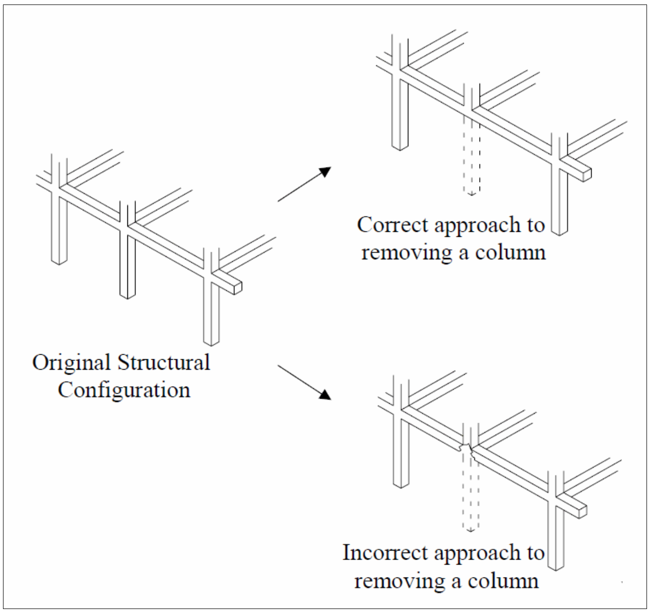

Figure 2.7: Representation of the "immaculate removal" approach. The same picture, with slight variations, is used in DoD UFC 4-023-03 and in GSA 2003. (Source: [15]).

Section B-4.2.2 is about the increase factor in the load combination (2.7).
"The factor of 2.0 acting on the Dead, Live and Snow Loads in Section 3-2.4.2 is used to account for the localized inertial effects due to the loss of vertical support over a short, finite period of time. The factor 2.0 is used in GSA 2003 and has been validated as conservative through a number of numerical simulations of progressive collapse."

Section B-4.2.3 is about the redistribution of the loads associated with "failed" elements.
"*When an element fails, the element's load must be transferred to the rest of the structural model. For Nonlinear and Linear Static analysis, the loads applied above the removed column or wall are doubled to account for inertial effects that can't be represented in a static solution. As these loads are already increased by a factor of 2.0, they are redistributed, without increase, to the structure below, over an area that is equal to or smaller than the loaded area that the failed element was supporting."*

"For Nonlinear Dynamic analysis, the entire structure is loaded as detailed in Section 3-2.4.1, without the factor of 2 that is used in Static analysis for the structural areas above the removed element. However, for Nonlinear Dynamic analysis, the loads from a failed element are doubled before being applied to the area below, to grossly account for the effect of structural elements falling upon other elements. The load is applied instantaneously, as is the removal of the structural element, which may induce significant dynamic response in the structure. The choice of 2.0 is based on engineering judgment. While the peak loads in a perfect impact will be much higher than 2.0, it is unlikely that elements will fail completely and fall intact upon the lower level. It is more likely that the element will be partially restrained, e.g. with rebar that is still embedded in the concrete, shear connectors between floor systems and beams, non-load-bearing walls, and other non-structural elements."

Examples of design and analysis for reinforced concrete, steel and wood are presented to illustrate tie force and Alternate Path calculations. The examples use the software SAP 2000NL.

Observations

Despite the fact that the requirements of DoD 2005 are said to be threat-independent, section B-4.1 states that the main motivation to require Alternate Path analyses "*is that DoD facilities could be attacked with artillery, rockets, mortars, or rocket propelled grenades"*. Furthermore, in the same section it is stated that "*however, it should be emphasized that the AP method is not intended to replicate an actual event*"

The presented procedures are more detailed than those of GSA 2003 (section 2.2.2.5).

The 2005 version of DoD UFC 4-023-03 is not in force anymore; the 2009 version introduced several changes, including different removal criteria of load-bearing elements, different increase factors and removed Damage limits in the Alternate Path analysis (Damage propagation is not admitted anymore).

2.2.2.4 DoD 2009

The 2009 version of UFC 4-023-03 (in the following referred as DoD 2009) "*is a significant revision to the 25 January 2005 version.*" The reasons for the changes include the incorporation of new knowledge, the resolution of some contradictions in terminology for structural concepts, and the clarification of ambiguities and imprecise guidance for linear static, nonlinear static, and nonlinear dynamic structural analysis methods.

Some of the significant changes are:
- replacement of Levels Of Protection (LOP) with Occupancy Categories (OC), to determine the required level of Progressive Collapse design;
- revision of the tie force method, including force magnitudes and locations of tie forces;
- inclusion of different increase factors in the load combinations of the Alternate Path analyses;
- Damage propagation in Alternate Load Path analyses is not admitted anymore;
- introduction of the "Enhanced Local Resistance", which is basically a hybrid between the Specific Load Resistance and the Indirect Design Method.

Occupancy Category	Design Requirement
I	No specific requirements
II	Option 1: Tie Forces for the entire structure and Enhanced Local Resistance for the corner and penultimate columns or walls at the first story. **OR** Option 2: Alternate Path for specified column and wall removal locations.
III	Alternate Path for specified column and wall removal locations; Enhanced Local Resistance for all perimeter first story columns or walls.
IV	Tie Forces; Alternate Path for specified column and wall removal locations; Enhanced Local Resistance for all perimeter first and second story columns or walls.

Table 2.3: The design requirements of DoD 2009 as a function of the Occupancy Category. (Source: [13]).

DoD 2009 prescribes that "*for new and existing buildings, all portions that are three stories or more shall be designed to avoid progressive collapse*".

The Alternate Path method "*is used in two situations: 1) for Option 1 of Occupancy Category II and for Occupancy Category IV, when a vertical structural element cannot provide the required tie strength, the designer may use the AP method to determine if the structure can bridge over the deficient element after it has been notionally removed, and 2) for Occupancy Category II Option 2, Occupancy Category III, and Occupancy Category IV, the AP method must be applied for the removal of specific vertical load-bearing elements which are prescribed in Section 3-2.9.*"
"***3-2.9.2 Location of Removed Load-Bearing Elements.***
For each plan location defined for element removal, perform AP analyses for:
1. First story above grade
2. Story directly below roof
3. Story at mid-height
4. Story above the location of a column splice or change in column size"
It must be noticed that DoD 2005 required that the analyses to be performed for every story of the building, unless "*the designer can show that similar structural response is expected for column removal on multiple floors*".

Detailed procedures for linear static, nonlinear static and nonlinear dynamic Alternate Path analyses are given.
The use of the linear static procedure is limited to structures that meet specific requirements for irregularities and Demand-Capacity Ratios (DCR). (A DCR is the ratio between the calculated internal forces and the expected strength of an element.)

For linear static and nonlinear static analyses, the gravity load combination for the bays immediately adjacent to the removed element and at all floors above the removed element is:

$$G = \Omega\ [(0.9 \text{ or } 1.2)\ D + (0.5\ L \text{ or } 0.2\ S)] \qquad (2.8)$$

which is similar to (2.7), with the difference that in (2.8) the increase factor Ω can assume values other than 2.0. The value of Ω are tabulated as a function of materials (steel, reinforced concrete, masonry, wood or cold-formed steel), structure type (framed or load-bearing wall), type of analysis (linear static, nonlinear static) and, for linear analysis, type of internal actions (force-controlled or deformation-controlled).
For floor areas away from removed column or wall, the load combination (2.6) remains valid.
Furthermore a fraction of the vertical loads must be applied horizontally in four separate analyses, i.e. one for each principal direction of the building.

Enhanced Local Resistance (i.e., an indirect version of the Specific Load Resistance mitigation strategy) is introduced for the cases listed in table 2.3. The number of elements for which it is required increases with the Occupancy Category.

In the commentary of DoD 2009, several observations are given.

The "three story requirement" is based on a minimum threshold of 12 estimated causalities:
"*C-2.1 Three Story Requirement and Story Definition.*
The required minimum height of 3 stories for progressive collapse design is taken from the original DoD guidance (DoD 2001). This requirement was based on a minimum threshold of 12 casualties

in a progressive collapse event where it was assumed that the 2 bays on either side of a removed column or wall would collapse on each of 3 floors and that each bay/room would house 2 persons. Thus, the justification for setting the limit at 3 stories was determined by the level of casualties and not by the mechanics of progressive collapse as a function of structural characteristics."

The incorporation of different increase factors Ω is discussed:
"*C-6.8 Load and Dynamic Increase Factors.*
Three analytical procedures may be employed: Linear Static, Nonlinear Static, and Nonlinear Dynamic. As progressive collapse is a dynamic and nonlinear event, the applied load cases for the static procedures require the use of load increase factors or dynamic increase factors, which approximately account for inertial and nonlinear effects. For both Linear Static and Nonlinear Static, the 2005 UFC 4-023-03 and the GSA Guidelines use a load multiplier of 2.0, applied directly to the progressive collapse load combination.
Three issues with the use of a fixed factor of 2 have been identified. First, the same load multiplier is used for Linear Static and Nonlinear Static analyses, although the Nonlinear Static analysis incorporates nonlinearity. Second, an increase factor of 2.0 is not appropriate for the majority of LS and NS cases. The maximum dynamic displacement of an instantaneously applied and sustained load in a linear analysis is twice the displacement achieved when the load is applied statically. If a structure is designed to remain elastic, a factor of 2.0 would be appropriate. However, in extreme loading events, it is typical to design structures to respond in the nonlinear range. Thus, the dynamic increase factor (DIF) that allows a Nonlinear Static solution to approximate a Nonlinear Dynamic solution, is typically less than 2. On the other hand, the load increase factor (LIF) for a Linear Static analysis must be greater than 2, since dynamic and nonlinear effects are present. Third, the load enhancement factor did not vary with the structural performance level, i.e., a structure is assigned a load enhancement factor of 2.0 regardless of whether the designer wants to allow significant structural damage or very little damage."

Damage spread is not allowed anymore in Alternate Path analyses:
"*C-6.9 Structural Damage Limits.*
In the previous UFC, the structural damage limits were set at 15% and 30% for the floor area above the removed column or wall at an external or internal column or wall, respectively. In this UFC, no damage to the floor is allowed and these criteria have been removed, as the floor system, beams, and girders in the bays directly above the removed column can be designed to not fail, as is done for the bays in the floors above the removed column location."
"*E-4.2.9 Iterate Dynamic Analysis.*
It is important to check that both stages of every analysis case converge. If the analysis does not converge, there is a problem with the model and it must be fixed. The problem could be numerical with assumptions made in SAP, but the

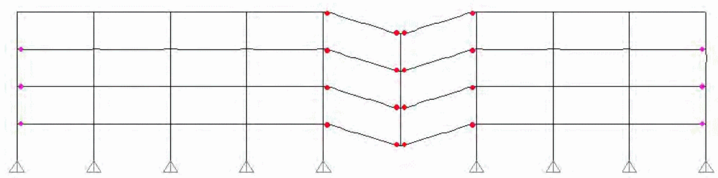

Figure E-18. Column 1 Removal Failed Convergence

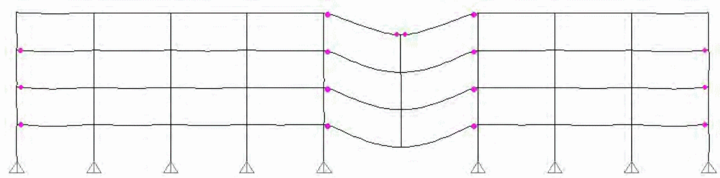

Figure E-19. Column 1 Removal Convergence After Redesign

Figure 2.8: Drawing from one of the examples included in DoD 2009 for Alternate Path analysis. In the first version of the structure a mechanism forms, so the analysis is immediately terminated and the structure is redesigned (Source: [13]).

most likely reason is that the model has a plastic hinge that failed or a mechanism has formed. At this point, the model cannot support the load. If the analysis fails to converge [...] members must be redesigned."

2.2.2.5 GSA 2003

The General Services Administration Guidelines "*Progressive Collapse Analysis and Design Guidelines for New Federal Office Buildings and Major Modernization Projects*" (GSA 2003) are meant to be used by for all new facilities or building modernization projects of the GSA. A first version of the Guidelines was released in 2000, and focused primarily on reinforced concrete structures; the 2003 version also addresses steel frame structures.

The Guidelines use a threat independent methodology and rely mainly on the Alternate Load Path method (referred as "Alternate Analysis Techniques").

A series of flowcharts are provided to guide the designer and in deciding if the building needs to be designed against Progressive Collapse or not. The flowchart criteria are based on use, occupancy, type of the building, standoff distance from moving or parked vehicles, as well as structural features such as seismic design.

If design against Progressive Collapse needs to be considered, then static or dynamic, linear or nonlinear analyses can be performed (although static analyses are not recommended for buildings taller than 10 stories). If the analysis results don't comply with given analysis criteria, then "*the facility exhibits a high potential for progressive collapse*" and shall be redesigned.

The overall recommended design strategy is to notionally instantaneously remove one primary vertical structural element (one column in frame structures, one structural bay or 9 m of wall, whichever is less, in load bearing wall structures), and to show by analysis that the resulting Damage is limited to:
- the structural bay directly associated with the removed element, or
- 170 m^2 (1800 ft^2) at the floor directly above (if the removed element is exterior), or
- 330 m^2 (3600 ft^2) at the floor directly above (if the removed element is interior).

In typical structural configurations, the analyses must be performed for the instantaneous loss of elements located:
- at or near the middle of the short side of the building;
- at or near the middle of the long side of the building;
- at the corner of the building.

If there is an uncontrolled ground floor area and/or an underground parking area, then the analysis must also be performed for the instantaneous loss of one internal element that extends from the floor of the area to the next floor (1 story). Additional analysis cases should be considered if there are significant changes in column or other load bearing member strength or configuration along any portion of the facility.
For structural configurations with an atypical structural arrangement, engineering judgment must be used to determine critical analysis scenarios that should be assessed, in addition to the situations previously listed. Possible structural configurations that may result in an atypical structural arrangement include, but are not limited to: combination structures; vertical discontinuities/transfer girders; variations in bay size/extreme bay sizes, plan irregularities; closely spaced columns.

The vertical load combination to be applied for static analyses is:

$$\text{Load} = 2(\text{DL} + 0.25\text{LL}) \tag{2.9}$$

and for dynamic analyses is:

$$\text{Load} = \text{DL} + 0.25\text{LL} \tag{2.10}$$

where DL is the dead load and LL is the live load.

The coefficient 2 in the static analyses combination (2.9) accounts for the dynamic effects due to instantaneous removal. If dynamic analysis is performed, member removal should take place in less than 1/10 of the period associated with the structural response mode of the structure.
It is also prescribed that "*the vertical element removal shall consist of the removal of the vertical element only. This removal should not impede into the connection/joint or horizontal elements that are attached to the vertical element at the floor levels*." This corresponds to the so called "immaculate removal" approach (figure 2.7).

The acceptance criteria for nonlinear analyses are given values of ductility and rotation limits for members. For static linear elastic analysis, the demand-capacity ratio (DCR) is used, defined as

$$DCR = Q_{UD}/Q_{CE} \tag{2.11}$$

where
Q_{UD} is the acting force on structural member or joint, and
Q_{CE} is the expected ultimate, unfactored capacity.

The allowable DCR values for the structural elements are:
- DCR < 2.0 for typical structural configurations
- DCR < 1.5 for atypical structural configurations

A step-by-step procedure, only for the linear elastic, static analysis is given:

"Step 1. *Remove a vertical support from the location being considered and conduct a linear-static analysis of the structure as indicated in Section 4.1.2.2. Load the model with 2(DL + 0.25LL)."*

Step 2. *Determine which members and connections have DCR values that exceed the acceptance criteria. If the DCR for any member end connection is exceeded based upon shear force, the member is to be considered a failed member. In addition, if the flexural DCR values for both ends of a member or its connections, as well as the span itself, are exceeded (creating a three hinged failure mechanism), the member is to be considered a failed member. Failed members should be removed from the model, and all dead and live loads associated with failed members should be redistributed to other members in adjacent bays."*

"Step 3. *For a member or connection whose* Q_{UD}/Q_{CE} *ratio exceeds the applicable flexural DCR values, place a hinge at the member end or connection to release the moment. This hinge should be located at the center of flexural yielding for the member or connection. Use rigid offsets and/or stub members from the connecting member as needed to model the hinge in the proper location. For yielding at the end of a member the center of flexural yielding should not be taken to be more than ½ the depth of the member from the face of the intersecting member, which is usually a column)."*

"Step 4. *At each inserted hinge, apply equal-but-opposite moments to the stub/offset and member end to each side of the hinge. The magnitude of the moments should equal the expected* *flexural strength of the moment or connection, and the direction of the moments should be consistent with direction of the moments in the analysis performed in Step 1."*

"Step 5. *Re-run the analysis and repeat Steps 1 through 4. Continue this process until no DCR values are exceeded.* ***If moments have been re-distributed throughout the entire building and DCR values are still exceeded in areas outside of the allowable collapse region, the structure will be considered to have a high potential for progressive collapse."***

Observations

The GSA Guidelines use a threat independent approach, "*as it is not feasible to rationally examine all potential sources of collapse initiation*." However the choice of the elements to be removed (single external vertical elements at the ground floor, plus one internal element if there is an uncontrolled ground floor area and/or an underground parking area) suggest that consideration was given mainly to vehicle-related Hazards (either vehicle collision or car bombing). This is also corroborated by the importance given to the standoff distance in the criteria to decide if the building needs to be analyzed.

The proposed analysis methodology does not consider more than one single element as initial Damage. This is justified in the introduction of the Guidelines: "*The approach taken (i.e., the removal of a column or other vertical load bearing member) is not intended to reproduce or replicate any specific abnormal load or assault on the structure. Rather, member removal is simply used as a "load initiator" and serves as a means to introduce redundancy and resiliency into the structure*." Furthermore, like in the DoD UFC 4-023-03, the "immaculate removal" approach is used.

It must be noticed that the DoD UFC 4-023-03 uses similar criteria to choose the elements to be removed, but it requires that the analysis must be repeated on several (DoD 2009) or even all floors (DoD 2005), not only on the ground floor.

The coefficient 2 in the the load combination for static analyses (2.9) accounts for the dynamic effects due to instantaneous removal. DoD UFC 4-023-03 uses a similar approach, but the increased load is only applied in the areas surrounding the removed element, i.e. where the dynamic effects really should arise. Other aspects of this approach are further discussed in section 2.2.2.4.

2.2.3 Survey of prescriptions

As a general overview, the following tables (taken from [35]) summarize the prescriptions of several regulations about Progressive Collapse. Some of the listed regulations are not in force anymore, but are included for comparison.

The surveyed regulations are the following:

- British Standard BS 5950-1:2000, Structural Use of Steelwork in Building
- BS 5628-1:1992, Code of Practice for Use of Masonry
- BS 5268-2:2002, Structural Use of Timber
- BS 8110-1:1997, Structural Use of Concrete
- BS 8110-2:1985, Structural Use of Concrete (Special Circumstances)
- National Research Council of Canada, National Building Code of Canada (NBCC) (1975)
- National Research Council of Canada, National Building Code of Canada (NBCC) (1977)
- National Research Council of Canada, National Building Code of Canada (NBCC) (1990)
- New York City Building Code (1998)
- New York City Department of Buildings, World Trade Center Building Code Task Force (2003)
- Department of Defense (2003) Unified Facilities Criteria (UFC) 4-010-01 Minimum Antiterrorism Standards for Buildings
- Department of Defense (2005) Unified Facilities Criteria (UFC) 4-023-03, Design of Buildings to Resist Progressive Collapse
- General Service Administration (GSA) Progressive Collapse Analysis and Design Guidelines

for New Federal Office Buildings and Major Modernization Projects (June 2003)
- American Society of Civil Engineers (1998), ASCE 7-98
- American Society of Civil Engineers (2002), ASCE 7-02
- American Society of Civil Engineers (2005), ASCE 7-05
- Interagency Security Committee (ISC) Design Criteria for New Federal Office Buildings and Major Reorganization Projects (2001)
- Precast Concrete Institute (PCI) Committee on Precast Concrete Bearing Wall Buildings (1976)
- Swedish Board of Housing, Building and Planning (Boverket, June 2000); Design Regulations BKR: Mandatory Provisions and General Recommendations, BFS 1993:58 with amendments up to BFS 1998:39, BFS 1999:7 and BFS 1999:46
- Swedish Board of Housing, Building and Planning (Boverket, 1994): Handbook on Vibrations, Induced Deformations and Accidental Loads
- Eurocode 1 - Section 2 – Actions on Structures, Part 1 – Basis of Design (CEN 250 1994) (pre EN 2002)
- Eurocode 2 – Design of concrete structures, Part 1, (prEN 1992-1-1: July 2002): General rules and rules for buildings

Table 2.4 summarizes the maximum extent of Damage propagation admitted by the building codes. In some cases (British Standards, National Building Code of Canada, New York City Building Code, World Trade Center Building Code Task Force) this extent defines the threshold between "local" and "disproportionate" or "progressive" Collapse.
In the case of DoD UFC 4-023-03 2005 and GSA 2003, the reported extent is the permitted "Damage limit" for the spread of Damage, as determined by structural analysis, resulting from the notional removal of one vertical load-bearing element (i.e., in an Alternate Load Path analysis). It must be remembered that the 2009 version of DoD UFC 4-023-03 does not admit Damage propagation in the Alternate Load Path analysis.

BS 5950-1: 2000	**Canada -NBCC 1977**	**NYC 1998, 2003**	**DOD UFC 4-023-03 2005**	**GSA 2003**
Horizontal Spread				
Lesser of 15 % of floor or roof area or 100 m^2 (1000 ft^2).	Truss, beam, floor strip or floor panel of initial damage plus one same on either side; one bay; two bay-sized slabs may hang together as a catenary if support at one end of slab is removed.	Lesser of 20 % of floor or roof area or 1000 ft^2 (100 m^2).	*Exterior*: Damage to floor above lost member shall be lesser of 70 m^2 (750 ft^2) or 15 % of total floor area; *Interior:* Lesser of 140 m^2 (1500 ft^2) or 30 % of total floor area. Damage must not spread beyond structure tributary to failed element (exterior) or beyond the bays adjacent to removed element (interior).	The structural bay associated with the removed member.
Vertical Spread				
Level of initial damage, plus one adjacent level, either above or below.	Level of initial damage, plus one adjacent level, either above or below.	$\leq$ 3 stories	Floor directly beneath failed element should not fail	1800 ft^2 (170 m^2) at the floor directly above a removed *exterior* column; or 3600 ft^2 (330 m^2) at the floor directly above a removed *interior* column.

Table 2.4: Maximum extent of Damage propagation admitted by some building codes. The 2009 version of DoD UFC 4-023-03 does not admit any Damage propagation in Alternate Load Path analyses. (Source: [35]).

Some of the regulations that contain provisions for structural integrity apply these provisions to all buildings by default. Other regulations recommend that Progressive Collapse only needs to be considered in determined cases, like for example in buildings that are above a certain height, or whose failure could cause severe loss of human life. Table 2.5 summarizes the threshold for consideration of Progressive Collapse.

British Standards Steel Concrete Masonry Timber	 All buildings All Buildings Buildings ≥5 stories Buildings ≥5 stories
Department of Defense – UFC 4-010-01	Buildings ≥3 stories
Eurocode 2002	**Consequence Classes 1 to 4:** 1) Low: 1 to 3 stories. No consideration. 2) Medium: 3 to stories, offices < 4 stories: Eurocode robustness and stability rules. 3) High: 7 to 10 stories, public buildings < 200 m^2: Simplified static analysis, prescriptive detailing rules. 4) Severe: > 10 stories, public buildings >200 m^2: Dynamic, nonlinear analysis, load-structure interaction.
Swedish Regulations	**Safety Classes 1 to 3:** 1) Little risk of serious injury. No consideration. 2) Some risk of serious injury. Consider only in multi-story buildings 3) Great risk of serious injury. Mandatory consideration.
GSA Guidelines (2003)	Exemption flowcharts regarding use, occupancy, building type, proximity of moving or parked vehicles, seismic design, and others.
Precast Concrete Institute (1976)	Horizontal ties in all buildings. Vertical ties in buildings over two stories

Table 2.5: Threshold for consideration of Progressive Collapse according to some building codes. (Source: [35]).

Table 2.6 reports the mitigation strategies prescribed by various building codes. The first row of the table (labeled “Risk”) mainly corresponds to the Event Control mitigation strategy (which is introduced in section 2.1.1 of the present work). The second row (“Layout”) corresponds to the Indirect Design Method (section 2.1.4). The third row reports provisions for the Alternate Load Path strategy (section 2.1.3). The fourth row corresponds to the Specific Load Resistance strategy (section 2.1.2); the “Key Elements” are those that need to be designed to resist exceptional loads. The fifth row reports prescriptions for the Indirect Design Method (section 2.1.4).

	BS Steel 2000	BS Concrete 97, 85	BS Masonry 1992	Eurocode 1-1994
Risk		Provide protection against impact.		Avoid, eliminate, or reduce hazards.
Layout		Avoid weakness in building layout.		Select structural form with low sensitivity to hazards considered. Avoid structural systems that may collapse without warning.
Direct Design: Alternate Load Path	Design building to resist notional removal of structural member, one at a time.	Design building to resist notional removal of structural member, one at a time. Length of wall considered lost is single load-bearing element taken as length between adjacent lateral supports or between lateral support and free edge.	Option 1: Building should not collapse following removal, one at a time, of vertical and horizontal elements, unless protected.	Select structural form that can survive accidental removal of an individual element, a limited part of the structure, or the occurrence of acceptable localized damage.
Direct Design: Key Elements	Design selected members as key elements.	Design selected members as key elements.		No recommendation
Indirect Design: Ties	Tie building together.	Provide horizontal ties around periphery, internally, to columns and walls. Provide vertical ties.	Option 2: Provide horizontal ties, but no vertical ties. Option 3: Provide horizontal and vertical ties.	Tie structure together.

Table 2.6 (part 1 of 4): Mitigation strategies prescribed by various building codes. (Source: [35]).

	Canada 1975	Canada 1977	Canada 1990	ASCE 7-98
Risk	Prevent storage of gas or explosives. Provide fenders against vehicles.	Reduce probability of occurrence of an abnormal event.	Control accidental events.	Design for extraordinary events (10^{-6} to 10^{-4} per year or greater).
Layout	Proper layout of walls and columns. Add spine walls.		Build in planes of weakness to arrest collapse propagation.	
Direct Design: Alternate Load Path	Design for possible change of direction of span of floor slabs; Strengthen internal partitions; Design slabs for catenary action; Reinforce walls to act as webs of beams.	Design for alternate paths, and possible change of direction of span of floor slabs; Strengthen internal partitions; Design slabs for catenary action; Reinforce walls to act as webs of beams.	Design building to resist notional removal of structural member.	
Direct Design: Key Elements	Design key elements for abnormal loads.	Design key elements for abnormal loads.	Identify and design key components to resist accidents.	
Indirect Design: Ties	Use energy-absorbing connections. Avoid joints that rely on friction due to gravity only.	Provide horizontal ties around periphery, internally, to columns, walls and around openings. Provide vertical ties. Use ductile connections. Brace trusses or connect them by diaphragms into groups.	Tie building together with horizontal, vertical and peripheral ties.	Provide continuity, redundancy and ductility in structure. Design ties for 20 kN/m (1400 lb/ft). Use non-linear, plastic analysis. Allow catenary or membrane action.

Table 2.6 (part 2 of 4).

	ASCE 7-02 and 05	DOD UFC 4-010-01 2003	DOD UFC 4-023-03 2005	ISC 2001
Risk	Design for extraordinary events (10^{-6} to 10^{-4} per year or greater).	Maximize standoff distance.	Refers to UFC 4-010-01	
Layout	Reduce span of long sections of crosswalls.	Structurally isolate additions to existing buildings, or inhabited from uninhabited buildings. Avoid building overhangs with inhabited spaces above them.	Refers to UFC 4-010-01	
Direct Design: Alternate Load Path	Allow local failure, but provide alternate load paths. Reinforce floor slabs for possible catenary action and change of span direction. Provide load-bearing interior partitions. Consider beam action of walls.	Refer to ASCE 7. Buildings should be able to withstand removal of one primary exterior or interior vertical or horizontal load-carrying element without collapse.	Applied to facilities designated as MLOP and HLOP as alternative to ties; withstand removal of vertical load-bearing element. Applied to facilities designated as LLOP when vertical tie requirements cannot be met.	Design new buildings to withstand loss of an exterior column for one floor above grade without collapse. Use symmetrical reinforcement.
Direct Design: Key Elements	Provide sufficient strength to resist failure from accidents or misuse. Resist load reversal.	Design all exterior (interior also if there is parking underneath) load-carrying columns and walls for an additional story height of unsupported length.	Apply UFCs for design to resist explosive effects; must still meet progressive collapse requirements.	
Indirect Design: Ties	Provide minimum levels of strength, continuity, and ductility. Tie principal elements of structural system. Provide returns* on walls. Use redundant structural systems, ductile detailing, and compartmentalized construction.	Provide continuity, redundancy and ductility in structure. Detail members to accommodate large displacements without loss of strength. Do not use URM as external walls.	For facilities designated as VLLOP provide horizontal ties; for LLOP provide horizontal and vertical ties. For MLOP and HLOP, provide horizontal and vertical ties or use alternate path method.	Use seismic, ductile detailing, and inelastic or post-elastic design. Dynamic analysis may be required. Shear reinforcement to allow large deformation. Grout or reinforce CMU. Minimize column spacing (< 30 ft or 9 m) and floor height (< 16 ft or 5 m).

* A return is a short length of wall usually at right angle to another wall.

Table 2.6 (part 3 of 4).

	GSA 2003	NYCBC 1998	NYC-WTCTF 2003	Sweden 1994
Risk				Accept severe damage to structure, but design to minimize risk to humans.
Layout				
Direct Design: Alternate Load Path	Notionally remove one column, one structural bay or 30 ft (9 m) of wall and analyze structure. For static, linear elastic analysis, limit DCR to 2 (typical construction) or 1.5 (atypical construction.) For nonlinear analysis, use ductility and rotation acceptance criteria. For dynamic analysis, remove member in less than 1/10 of period of corresponding structural response.	Notionally remove structural element: -one column; -one wall panel; -two adjacent wall panels; -any vital one element and check for progressive collapse.	Refer to ASCE 7. Notionally remove key members one at a time and assess stability of structure under D + 0.25 L + 0.2 W_n.	Bridge local damage. Allow large deformations and membrane effect. Apply plasticity and ultimate strength theory.
Direct Design: Key Elements	Design all exterior columns between 1st and 3rd floors, and all columns in public spaces, for an additional story height of unsupported length.	Check key members and connections for a pressure of 5 psi (34 kPa).	Harden locally key elements, transfer structures, and column connections that support transfer structures. Detail to resist load reversals.	
Indirect Design: Ties	Use redundant lateral and vertical elements and detail for continuity and ductility. Design against load reversal and shear failure.		Tie building together. Transfer structures shall be continuous over several supports. Column connections that support transfer structures shall be full moment connections capable of providing sustained strength despite inelastic deformations.	Design floors to resist falling debris; floors and bearing walls shall be able to transfer a tension of 20 kN/m (1400 lb/ft) and a shear of 20 kN/m.

Table 2.6 (part 4 of 4).

Table 2.7 reports the load combinations prescribed by various building codes for checking structures. The first column ("Load combinations after notional member removal") corresponds to the Alternate Load Path strategy, while the third column ("Accidental load") corresponds to the Specific Load Resistance strategy.

Standards	Load combinations after notional member removal	Accidental load
BS	$(1\pm0.5)\,D + L/3 + W_n/3$	34 kPa (5 psi)
Eurocode 2003 draft		20 kPa (3 psi)
Canada 1977	$D + L/3 + W_n/3$	
ASCE 7-98, 02, 05	(0.9 or 1.2) D + (0.5 L or 0.2 S) + 0.2 W_n (with member removal) 1.2 $D + A_k$ + (0.5 L or 0.2 S) (specific local resistance method) (0.9 or 1.2) $D + A_k + 0.2\,W_n$ (specific local resistance method)	A_k
DOD UFC 4-010-01	$D + 0.5\,L$ net floor uplift	
DOD UFC 4-023-03	$D + 0.5\,L$ net floor uplift (0.9 or 1.2) D + (0.5 L or 0.2 S) + 0.2 W_n (nonlinear dynamic analysis) 2.0 [(0.9 or 1.2) D + (0.5 L or 0.2 S)] + 0.2 W (static analysis)	
NYC 1998, 2003	$2\,D + 0.25\,L + 0.2\,W_n$	
GSA	$2\,(D + 0.25\,L)$ static analysis $D + 0.25\,L$ dynamic analysis	
Sweden	$G_k + \Psi Q_k$	Q_{ak}

D, L, W_n, S = dead, live, wind and snow loads;
Q_{ak} = characteristic value of accidental action;
G_k, Q_k = characteristic dead, imposed loads per unit area of the floor or roof; Ψ is a load reduction factor which, when multiplied with Q_k, gives the frequent value of a variable action.
A_k = extraordinary load.

Table 2.7: Load combinations prescribed by various building codes for checking structures. (Source: [35]).

2.3 Summary and comments

In this chapter, section 2.1 lists and explains the four categories of strategies that have been identified for Progressive Collapse mitigation.
Section 2.2 presents a review of some of the most significant regulations about Progressive Collapse, including their historical evolution.
From this information several observations can be derived.

The level and type of attention given to Progressive Collapse has changed through the years. Between the late 1960s and the mid-1970s the "new phenomenon" Progressive Collapse attracted interest of the researchers and prompted changes in building codes. Decades later, some of these research results are still referenced nowadays, and some building codes requirements are still in force with little or no modifications.
The level of attention given to Progressive Collapse was especially boosted by three important cases: the Ronan Point Tower (which occurred in 1968; see section 1.4.1), the A.P. Murrah Federal Building (1995; section 1.4.3), and the World Trade Center (2001; section 1.4.4). In other years, less attention was given. A significant example is the case of the National Building Code of Canada, whose discussion about Progressive Collapse in the 1975 version is much longer than the 1995 version, because "*the extent of the discussion reflects the importance accorded to the topic at the time*"([35], section A.3.3).
The type of attention also changed. Most of the early studies referred mainly to accidental gas explosions as prompting events of the Collapses; likewise, the regulation were giving much attention to precast structures. Conversely, nowadays the attention is mainly on terroristic attacks.

Most of the currently available data about Collapses and exceptional Hazards are decades old and were collected in less than optimal ways (e.g. from news sources, rather than from investigations). For example the following quote, from Leyendecker and Burnett [23], is significant: "*the number of incidents causing damage in excess of a specific level (measured in dollars) or a brief description of damage is usually provided. There is rarely complete information on the type or number of structures affected or an adequate description of their damage. There is also no available US data which provide a correlation between a specified damage level and a corresponding damage loss in dollars. Finally there are no data on the load characteristics of the actual events*."
Furthermore, the incidence of some Hazards could have changed in the years (e.g. the probability of vehicular impact on buildings as a consequence of the increased number of vehicles).
It is likely that the amount and the quality of the information could be greatly improved, if a good methodology to collect it was devised and enforced.

The regulations could be improved. Some regulations generically request the designer to avoid or contain the propagation of local Damage, but give little or no guidance on how to do it.
Other regulations provide more detailed instructions, which basically consist in applying one or some of the strategies listed in section 2.1.
In these regulations, there often is not a real required performance. For example, the Alternate Load Path analysis "*is not intended to reproduce or replicate any specific abnormal load or assault on the structure. Rather, member removal is simply used as a "load initiator" and serves as a means to introduce redundancy and resiliency into the structure*" (GSA 2003 [15]).
With other types of Hazards it is generally assumed that, by applying the prescriptions of modern building codes, the probability that a given performance will not be fulfilled will be lower than an acceptable value. With Progressive Collapse the prescribed methodologies can apparently be similar, but by applying them we cannot be sure that the probability of having a given consequence (like the occurrence of a Collapse of a specified extent) will be lower than an acceptable value.

Furthermore, for some building code requirements and assumptions "*engineering judgment was used*" (DoD 2005 [12]) or, in other words, the used criteria are not objective.

The considerations here presented are an influence on the development of the methodologies proposed in chapter 5 of the this work.

Chapter 3 - Quantification of Progressive Collapse propensity

Several ideas have been proposed for parameters to quantify the propensity to Progressive Collapse of structures. In literature these parameters are generally referred with various names, such as "Robustness Index" or "Vulnerability". Such parameters would be useful to decide if a given structure is safe enough against Progressive Collapse, or if mitigation methods need to be applied. The present chapter lists and analyzes some of these ideas.

Section 3.1 clarifies some aspects about the terms "robust" and "Robustness".
Section 3.2 lists several of the ideas that have been proposed for the quantification of Progressive Collapse propensity, and elaborates on them.
Section 3.3 summarizes the chapter and elaborates on its concepts.

3.1 The concept of Robustness

The adjective "robust" and the noun "Robustness" are widely used in the literature about Progressive Collapse. They are also commonly used in many other fields of science and technology.

Several different definitions and meanings of "robust" and "Robustness" exist. In general, even inside the same field there can be multiple definitions and/or meanings. The common concept of all definitions is that in a robust system, a change in the input produces a small variation in the output. The smaller the variation, the more robust the system. As a consequence, the term "Robustness" can actually have several different meanings, even for the same system, because different input and output parameters can be considered.

For example, in the context of Progressive Collapse, the input parameters can be the intensity of a traumatic event, or the extension of the initial Damage, or others; the output parameters can be the occurrence of indirect Damage, the final extension of the Damage, short and long term monetary Losses, or others.

As a consequence, most of the ideas proposed in literature calculate "Robustness Indexes" that cannot be compared to each other, and a structure might be considered robust according to one index and non-robust according to another one. To avoid ambiguity, the three "components" of Robustness (system, input parameters and output parameters) should always be clearly stated. Section 3.2.1.1 describes a paper in which these concepts are further discussed.

3.2 Ideas that have been proposed to quantify Progressive Collapse propensity

The following sections describe several ideas that have been proposed for the quantification of Progressive Collapse propensity. Each idea is also commented. Section 3.3 summarizes the listed ideas and elaborates.

3.2.1 Maes et al.

The article "*Structural robustness in the light of Risk and consequence analysis*" [29], by Maes, Fritzons and Glowienka, elaborates on the concept of Robustness and proposes three Robustness indicators; one example is proposed, in which the three indicators are applied.

3.2.1.1 About Robustness

The paper first highlights that the meaning given to the word "robust" will likely vary from person to person. It quotes the Oxford dictionary, which defines "robust" as "*pertaining to, or requiring bodily strength or hardiness and vigour; possessing or indicating great strength*". Furthermore, it states that in civil engineering, it is used as a synonym for stability, ductility, reserve strength capacity, redundancy, or as an opposite of Vulnerability and fragility.

Then definitions of "Robustness" are given for different fields of science and technology: Control Theory, Statistical Inference, Bayesian Decision Making, Quality Control and Product, Development, Design Optimization, Ecosystems and Immune Systems and Software Engineering.

According to the Authors, "*the modern usage of "robustness" [...] refers to the manner in which certain performance objectives or system properties are affected by hazardous or extreme conditions*" and "*Robustness is also a measure of the persistence in time of certain qualitative features in a system in response to perturbations*".
Furthermore, "*it should be noted that robustness is defined for "specified performance objectives" of a given system, with "specified perturbations" being applied to the system. It makes no sense to speak of a system being robust without first specifying both the feature and the perturbations of interest*".
"*For civil engineering infrastructure, the "performance objectives" are almost always related to consequences (what happens after something goes wrong?). They can be broad, as in: system survival, post-disaster operational capability, limitation of financial losses, safety to people, sustainability, and minimal environmental impact; but they can also act as indirect objectives and therefore they can be narrow and geared towards concepts intrinsic to structural design, such as: maintaining sufficient redundancy, ductility and reserve capacity, or the containment of very specific consequences or followup consequences*".
"*It should be noted that the aspect of insensitivity to perturbations includes the common (but narrower) definition of robustness as the ability of a system to sustain damage following an extreme disturbance. In this context, system survivability is the key performance objective and excessive consequences such as system failure or collapse are to be avoided*".

3.2.1.2 First proposed idea: maintaining sufficient system structural resistance

For the definition of the first Robustness indicator, it is considered a system consisting of n members which can be in safe or failed states. The assumed objective is to ensure that the system's resistance as a whole can be maintained to a sufficient degree following an environmental load that causes failure in one of its components i.
The reserve strength ratio (RSR) is defined as ratio of the environmental load at system Collapse divided by the original design environmental load.
The first Robustness indicator is defined as

$$R_1 := \min_i \frac{RSR_i}{RSR_0} \tag{3.1}$$

where RSR_i is the reserve strength ratio of the structure when the i-th of its elements is impaired ("failed"), and RSR_0 is the reserve strength ratio of the structure with no impaired elements.
The Authors highlight that the indicator R_1 gives no consideration to consequences of failure or hazard likelihoods.

3.2.1.3 Second proposed idea: maintaining sufficient system structural reliability

For the second Robustness indicator, the assumed objective is to maintain system reliability in a failed condition following the occurrence of a disturbance of a specified intensity.
The system failure probability of the undamaged system is notated as P_{s0}, while the system failure probability of with the i-th element impaired is notated as P_{si}.
The indicator is defined as

$$R_2 := \min_i \frac{P_{s0}}{P_{si}} \tag{3.2}$$

3.2.1.4 Third proposed idea: containing severe system consequences

The third Robustness indicator refers to the cases in which "*the system cannot easily be expressed as an assembly of components or members as in the case of a complex or a continuous structural system*". The indicator has been developed to be suitable "*to deal with the objective of containing the costs associated with the consequences of failure C_F in a system subject to an external hazard X*".
First the Authors show a diagram of hazard intensity versus consequences of failure (figure 3.1, left), in which sample response (a) shows a very robust structure, (b) shows a structure in which consequences and hazard are relatively proportional and (c) shows a system with very little Robustness. Then the probability of exceedance of failure consequences is obtained, given the occurrence of an extraordinary hazard, by integrating over the probability density function of the hazard. By plotting this probability in logarithmic scale (figure 3.1, right), "*a good idea of the tail behaviour of the failure consequence distribution can be obtained*". The resulting graph "*can be used qualitatively to rank system robustness. If a more quantitative ranking is desirable, then a suitable measure of robustness R_3 can be defined as the inverse of the tail heaviness H of the log-exceedance curve*"

$$R_3 := 1/H \tag{3.3}$$

It is highlighted that H is less than one for case (a), equal to one for case (b) and greater than one for case (c).

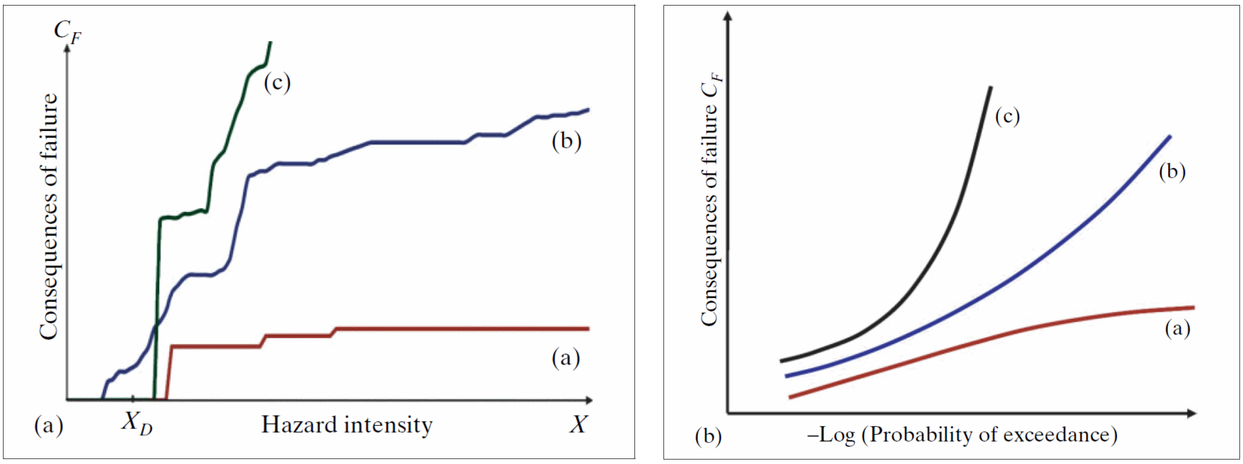

Figure 3.1: Third proposed idea for a Robustness indicator by Maes et al. (Left) Diagram of hazard intensity versus consequences of failure. (Right) Diagram of probability of exceedance of failure consequences versus consequences of failure. (Source: [29]).

3.2.1.5 Proposed example

An example of the application of the proposed ideas is presented. It consists of a chemical lab building, which contains toxic chemicals, in which forklifts operate. The hazard consists in severe forklift impact on the central wall of the building, which can cause Collapse of a floor slab. The performance objectives is the containment of the failure consequences, in terms of: life safety of the occupants, prevention of excessive economic Losses due to Damage in the laboratory and the surrounding buildings, environmental protection against spilling of chemicals and prevention of long-term and indirect Losses to the company. The behavior of the structure is modeled with plastic analysis.

The first indicator R_1 is easily calculated by comparing the ultimate load of the floor slab with and without the central wall, using characteristic strength values. The Authors observe that "*R_1 says nothing about the hazard itself. It serves only to contrast the intact with the impaired system*".

To calculate the second indicator R_2 a reliability analysis of the slab, with and without the central wall, is performed using the Second Order Reliability Method (SORM); the assumed probabilistic distributions and parameters are taken from the Joint Committee on Structural Safety (JCSS) Probabilistic Model Code [19] (figure 3.2). The Authors do not comment after the calculation of R_2.

Variable	Distribution	Mean m_x	V
Self weight	Normal	7,5 kN/m²	5%
Construction load	Normal	2,0 kN/m²	10%
Extreme live load	Gumbel	2,09 kN/m²* 1,86 kN/m²**	48%* 36,8%**
Uncertainty of load effect on the slab	Lognormal	1,0	20%
Concrete compression	Lognormal	38 MN/m²	13,2%
Yielding stress	Lognormal	560 MN/m²	5%
Slab height	Normal	0,30 m	2%
Concrete cover	Normal	0,025 m	20%
Uncertainty of slab resistance	Lognormal	1,2	15%

* based on a surface area 64 m² and life of 50 years
** based on a surface area 128 m² and life of 50 years

Figure 3.2: Table of the basic random variables used to calculate the indicators R_2 and R_3. (Source: [29]).

Next, the third indicator R_3 is considered. According to the Authors, "*the risk/consequence analysis involves extensive event trees in combination with structural reliability analysis*". A total of 25 random variables are included in the reliability analysis, which is performed "*based on the Eurocode 2 model-column-method*". The considered variables include those that characterize the exceptional hazard, like "*Occurrence of forklift traffic*" and "*Probability of more than one critical forklift traffic impact in 50 years*". The cost of the consequences is expressed in Euros (figure 3.3, left). In the end, a diagram of the consequences of failure as function of their probability of exceedance is obtained (figure 3.3, right). The Authors comment that "*it shows that*

Consequence	Costs (EUROS)	Including long term costs (EUROS)
Wall damage, removal, forklift damage etc.	50,000	–
Building damage (rebuilt)	900,000	–
1 additional building damaged, repair, production delay	8,900,000	–
2 additional building damaged, repair, production delay	18,150,000	–
1 fatality	1,500,000	–
1 unit of chemicals clean up	100,000	450,000
2 units of chemicals clean up	200,000	1,125,000
3 units of chemicals clean up	400,000	2,812,000
4 units of chemicals clean up	800,000	7,031,000

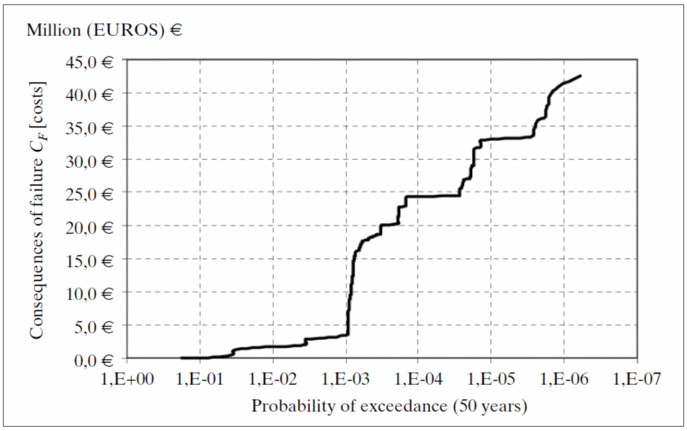

Figure 3.3: Example of application of the third proposed idea for a Robustness indicator by Maes et al. (Left) Table of costs of the consequences. (Right) Calculated diagram of probability of exceedance of failure consequences versus consequences of failure. (Source: [29]).

the structural robustness is quite sensitive to the immediate consequences of the slab failure as well as the volume of chemicals spilled"; a value of the parameter R_3 is not actually calculated.

Observations

The first proposed Robustness indicator R_1 follows a deterministic approach, while R_2 and R_3 follow a probabilistic approach. The Authors stress that the indicators differ for the different consideration given to hazard likelihood and consequences of failure (i.e, "input" and "output" parameters of Robustness; see section 3.1).
All the proposed ideas only consider one impaired element at a time (i.e., they do not consider the events in which multiple elements are impaired), even though they could easily be extended to include more cases.
It is stated that R_1 and R_2 can be used to ensure that the system's resistance and reliability can be maintained to a sufficient degree; no specific criterion is presented to quantify this sufficient degree. For R_3 the target is containing the costs associated with the consequences of failure; in the example, the estimated costs are represented by a diagram, rather than by parameter R_3 itself.
The reported example refers to one determined exceptional hazard (the forklift impact) and only one case of damaged structural element (the central wall). The approach for R_3 would likely work if several exceptional hazards were considered; it is not clear if it would work for multiple Damage scenarios.
Applying the idea for the third Robustness indicator R_3 requires a probabilistic characterization of the exceptional hazard, and in the presented example it likely required an important computational effort. The article does not specify many details about the used computation method.

3.2.2 Starossek

Starossek proposes five approaches for the quantification of a "Robustness index" or a "Collapse resistance index". The following information is taken from a thesis supervised by the Author himself [18], in which the ideas are presented, analyzed and commented by the Student. Some comments of the Student are also reported here.
The Author defines Robustness as a structure's insensitivity with respect to a local malfunction or to a local Damage. The Author also distinguishes between "Robustness" and "Collapse resistance". Collapse resistance is defined as the property of a structure of being insensitive to accidental circumstances. The difference between the two definitions is that Robustness only considers a local malfunction or Damage, while Collapse resistance also considers the events that can cause such malfunction or Damage.

3.2.2.1 First proposed idea: differential Damage

Given a structure, some local Damage is introduced and its reaction is observed.
The Robustness index RI is defined as the complementary of the normalized Damage:

$$RI:=1-\max_j(d_j) \tag{3.4}$$

where d_j is a normalized measure of the total Damage caused by the loss of the j-th structural restraint condition. Because of the normalization, both d_j and RI are comprised between 0 and 1.

Observations

The approach is deterministic. It is very general, in that it is not specified how the Damage is quantified and calculated. The considered input is the initial local Damage determined by the loss of restraints, while the considered consequences are the maximum final extension of the Damage. It is not specified how many cases of restraint loss need to be considered.
No example of the application of this approach is given. No criterion is given to define an

acceptable limit value of RI. Consistently with the given definition, the cause of the initial Damage is neglected.

3.2.2.2 Second proposed idea: integral Damage

This approach is considered by the Author as an extension of the previous one. The Robustness index RI is defined as:

$$RI := 1 - 2\int_0^1 [d(i) - i]\,di \qquad (3.5)$$

where i is a measure of the normalized initial Damage, and d(i) is a measure of the normalized final Damage. It is not specified how i and d(i) are quantified and calculated.
The value of the index spans between 1 for a perfectly robust structure to 0 for total lack of Robustness.
The index can be graphically represented as illustrated in figure 3.4 (left), in which the horizontal axis represents the normalized initial Damage i, the vertical axis represents the normalized final Damage d(i) and the areas comprised between one of the curves (A, B or C) and the diagonal of the square represent the values of RI for three different cases. The line labeled B represents a particularly robust structure, while the line labeled C represent a particularly non-robust structure. It is observed that line A and B should give values of RI in the same order of magnitude, even though structure B would be considered more robust than structure A according to the given definition of Robustness, because for lower values of the initial Damage the structure B has a smaller final Damage.

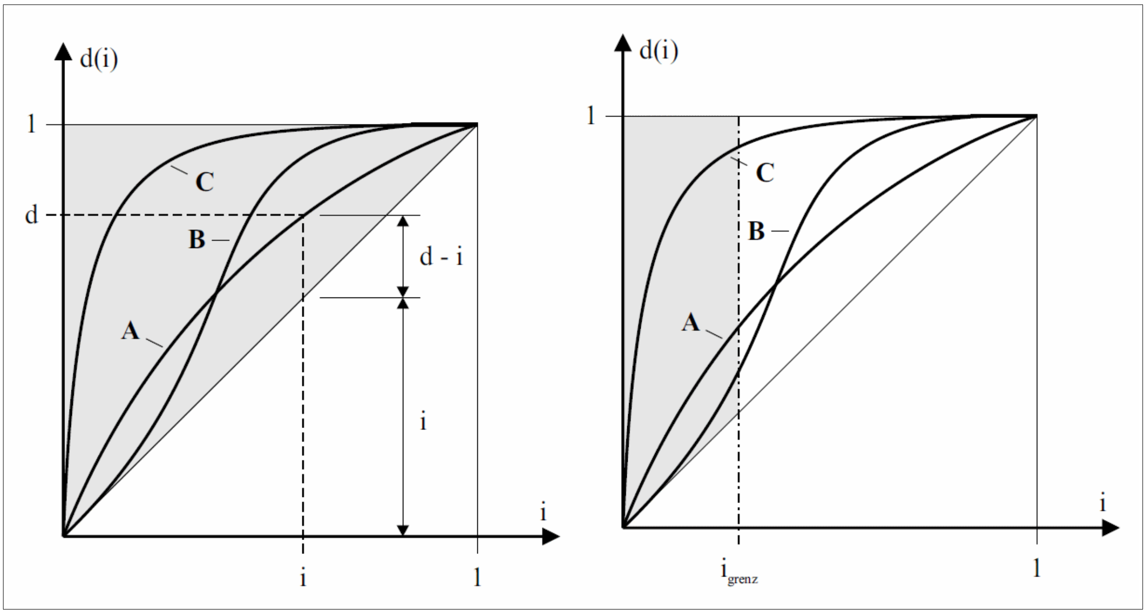

Figure 3.4: Second proposed idea for a Robustness index by Starossek. (Left) Graphical representation of definition (3.5). (Right) Graphical representation of definition (3.6). (Source: [18]).

It is also observed that, since the considered definition of Progressive Collapse includes the concept of disproportion between initial Damage and its consequences, and the considered definition of Robustness considers a local initial Damage, lower values of i should be more important than higher values. Thus, starting from the definition (3.5), the following new definition is formulated:

$$RI := 1 - \frac{2}{i_{grenz} * (2 - i_{grenz})} \int_0^{i_{grenz}} [d(i) - i] di \qquad (3.6)$$

in which i_{grenz} is the maximum considered value of the initial Damage.
This new definition is represented in figure 3.4 (right). Like in the previous definition, the value of the index spans between 1 and 0.

Three more observations are given.
One is that the integrals in (3.5) and (3.6) need to be discretized, since the parameters in the integrals cannot be determined continuously.
The second is that for each intensity i of the initial Damage there are different final Damage configurations j, which differ for type, location and shape of the Damage. Thus it is proposed to define the normalized final Damage as:

$$d(i) = \max_j d(i_j) \qquad (3.7)$$

The third given observation is that the proposed Robustness index is currently not calculable because of the big effort that it would require.

Observations
As with the first proposed idea, it is not specified how the Damage is quantified and calculated. The cause of the initial Damage is neglected and no criterion is given to define an acceptability limit value of RI. No example of the application of this approach is given.
With reference to figures 3.4, the slope angle of the curves for low values of i could be more meaningful than the subtended areas.

3.2.2.3 Third proposed idea: differential energy

This approach is based on the comparison of three types of energy:
- E_i (initial): energy necessary for the occurrence of an initial failure
- E_r (released): energy released through the initial failure
- E_s (subsequent): energy necessary for the failure of a subsequent element

It is observed that, in order to have Damage progression, the released energy must be bigger than than the energy necessary to damage a subsequent element:

$$E_r \geq E_s \qquad (3.8)$$

From this necessary (but not sufficient) condition a Robustness index is derived.
The condition (3.8) is only valid for failure of a second element. To take into account the progression of failure to other elements, the following generalization is considered:

$$E_{r,j} \geq E_{s,j+1} \qquad (3.9)$$

where $E_{r,j}$ is the energy released from failure of the j-th element; $E_{s,j+1}$ is the energy necessary for the failure of the (j+1)-th element; and $E_{r,0}$=E_r.
Complete progression of the Damage up to total failure does not occur if the following condition is respected and no external energy is given to the structure:

$$\sum_{k=0}^{j} E_{r,k} < \sum_{k=0}^{j} E_{s,k+1} \quad \text{with } j=1,...,n-1 \tag{3.10}$$

where n is the total number of elements of the considered Damage scenario.
From condition (3.10) the following definition of Robustness index is derived:

$$RI := \frac{\sum_{k=0}^{n} E_{s,k+1}}{\sum_{k=0}^{n} E_{r,k}} \tag{3.11}$$

If RI>1, then total failure does not occur; if RI≤1, then total failure is possible.
Since n is defined as the total number of elements of the considered Damage scenario, in the previous text line "*total failure*" should mean "*failure of all the elements of the considered final Damage scenario*", and not "*failure of all the elements of the structure*". It is highlighted that, in order to apply the proposed idea, all possible Damage scenarios must be investigated, which would require a very big effort.

Observations
This approach is deterministic. It is not stated how the energies can be calculated. The index is based on a necessary but not sufficient condition; as a consequence it should tell if a Collapse scenario is possible, not if it actually happens.

3.2.2.4 Fourth proposed idea: integral energy

This approach is aimed at measuring a Collapse resistance index (CRI). It's worth to remember that the Author defines "Collapse resistance" as the property of a structure of being insensitive to accidental circumstances; while the proposed definition of "Robustness" only considers a local malfunction or Damage, "Collapse resistance" also considers the events that can cause such malfunction or Damage.
The Collapse resistance index is based on the comparison of the energy necessary for the complete destruction of the structure E_d and its total mass or weight M:

$$CRI := \min E_d/M \tag{3.12}$$

Another version of the index is obtained by substituting the mass M with the potential energy of the structure prior to failure E_p (which is given by the structural masses multiplied by their elevation from a reference level):

$$CRI^* := \min E_d/E_p \tag{3.13}$$

Another proposed variant of the index is similar to (3.13), but substitutes the bounding energy E_b for the potential energy E_p.

Observations
The proposed indexes refer to the total Collapse of the structure, and it seems that they should not give information about partial Collapses. It is not stated how the energy necessary for the complete destruction of the structure can be calculated. No criterion is given to assess from the proposed indexes if the studied structure is to be considered acceptable.

3.2.2.5 Fifth proposed idea: stiffness

Two Robustness indexes are defined by comparing the stiffness of a structure in damaged and undamaged state.
The first index is calculated on the basis of an average of all damaged configurations:

$$RI := \left(\frac{1}{n}\sum_{i=1}^{n} K_i\right)/K_0 \tag{3.16}$$

the second index is calculated on the basis of the most unfavorable Damage scenario:

$$RI := \min_i (K_i)/K_0 \tag{3.17}$$

where n is the number of the elements or of the system restraints; $K_i=\det(\mathbf{K_i})$ is the determinant of the modified stiffness matrix, obtained by removing one element from the structure or one restraint from the model; and $K_0=\det(\mathbf{K_0})$ is the determinant of the stiffness matrix of the undamaged structure. The value of both indexes can vary within 0 and 1.

In [18] it is highlighted that these indexes are easier to calculate than the other ones, because they don't require solution of static or dynamic problems. However, they should provide less information than other ones because they do not consider Damage progression or impact of structural elements. It is also stated that, according to some test calculations, the indexes quickly assume very low values, thus a scaling of the parameters is suggested.

Observations
Again, no criterion is given to assess from the proposed indexes if the studied structure is to be considered acceptable. It is not taken into account that the entry values of a stiffness matrix is likely to change during a Collapse, because of the non-linear effects involved. For example, in order to model a catenary effect (section 2.1.3) geometrical nonlinearities must be considered. Thus the considered stiffness matrices could be not representative of the structure's actual behavior.

3.2.3 Giuliani et al.

The paper "*Strategie per il Conseguimento della Robustezza Strutturale: Connessione e Compartimentazione*" [17], (italian for "*Strategies to Achieve Structural Robustness: Connection and Compartmentalization*") by Giuliani and Wolff, gives two definitions of structural Robustness and proposes one algorithm to quantitatively estimate it.

The first definition is qualitative: Robustness is the ability of a structure to maintain an adequate level of structural integrity after a critical event that directly provokes failure of a localized part of the structure.
The second definition is quantitative: Robustness is defined as the ratio between an increase of the structural Damage level ΔD and the corresponding resistance decrease ΔR. Furthermore, a structure can be deemed robust if the ratio is lower than a limit value L:

$$\text{robust structure} \leftrightarrow |\Delta R|/\Delta D < L \tag{3.18}$$

The Damage level D is quantified as the number of failed structural elements. The limit value L must be chosen according to the importance of the structure and exposure to threatening events; the paper does not provide more specific criteria, nor examples, to quantify the limit value L.

The algorithm to calculate the Robustness of a structure is the following:
1. The structure is modeled, undamaged (D=0) and subjected to loads.
2. A nonlinear static analysis is performed, increasing the load. The ultimate resistance is given by the load multiplier λ that causes kinematic indeterminacy of the structure.
3. The Damage level D is increased by one unit.
4. D elements are removed from the structure and the static analysis is repeated.

Step 4 is reiterated for all the possible combinations of D elements removed.
Then the algorithm goes back to step 3, i.e. the Damage level is increased by one unit and step 4 is performed again for all the possible combinations of the new Damage level.
The algorithm stops at a determined Damage level.
The results of the analysis can be represented in a digram like the one in figure 3.5, in which the horizontal axis reports the Damage level D and the vertical axis reports the critical load multipliers λ. For each value of the Damage D, many values of the load multiplier λ are calculated; the blue and red lines in the diagram represent the maximum and minimum calculated values of λ, respectively. The structural Robustness, as defined, can be read from the diagram as the slope of the lower curve. Since the number of possible combinations quickly becomes very high, probabilistic optimization methods are used to identify the two most significant combinations, i.e. the ones that give maximum and minimum resistance.

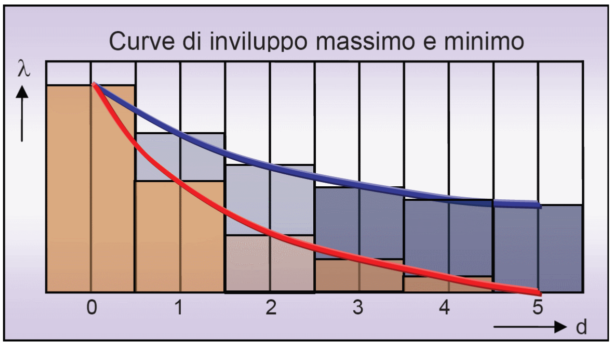

Figure 3.5: Idea for Robustness quantification by Giuliani et al. Diagram of Damage versus critical load multiplier. (Source: [17]).

Observations
The proposed method follows a deterministic approach. Robustness is defined in terms of loss of resistance as a consequence of a localized Damage. The resistance is quantified as the maximum applied load that causes kinematic indeterminacy.
Rather than a single value of structural Robustness, the proposed method gives one value for each increment of Damage level ΔD.
The paper gives a quantitative criterion to assess if a structure can be deemed "robust", by comparing the calculated ratio |ΔR|/ΔD with a limit value L (3.18), but only generic qualitative criteria are given to quantify L.
It must be observed that the slope of the Damage/lowest resistance curve might not be the best (or the only) parameter to look for in assessing if a structure is to bee deemed safe enough. For example, it might be more useful to look for the Damage level for which the resistance of the structure becomes lower than the design loads; this condition should correspond to the start of the propagation of the Damage (figure 3.6).

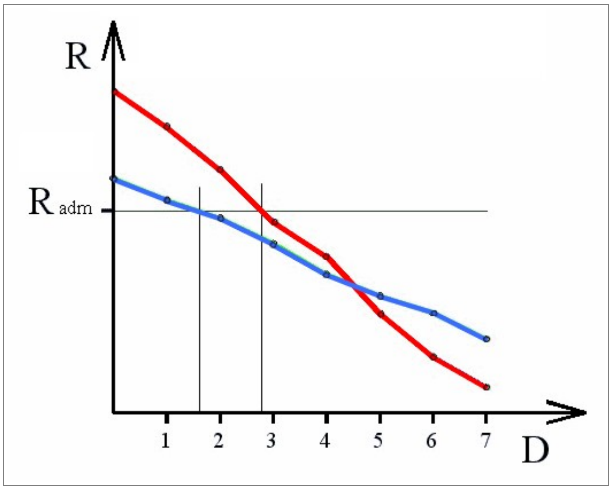

Figure 3.6: In this example, the structure represented by the red line would be deemed less robust than the one represented by the blue line, according to the definition by Giuliani et al. Yet, if R_{adm} is the load that the structures need to bear, the blue structure reaches the critical condition for a lower Damage level D.

3.2.4 Lind

The paper "*A Measure of Vulnerability and Damage Tolerance"* [25] by Lind, presents probabilistic definitions of "Vulnerability" and "Damage tolerance", which are considered complementary concepts and thus are expressed by reciprocal numbers. The paper looks on any type of system, including (but not limited to) structures subjected to Progressive Collapse.

Damage tolerance is defined as tolerance of Damage unforeseen or not considered in the design. The sinking of the Titanic and the Ronan Point Collapse (see section 1.4.1) are reported as examples lack of Damage tolerance.
Damage tolerance is also defined as tolerance of localized or minor harm to the system. Collapse of long electric transmission lines in "domino style" after one conductor snapped is reported as an example of the second definition.

The paper lists some qualities that are desirable in a quantitative measure of Vulnerability:
- it should be a decision-making tool for design or redesign;
- it should be calculable;
- it should be valid.

The meaning given to the word "valid" is:
- the measure actually expresses the tolerance of Damage and little else;
- it provides the ability to distinguish reliably between systems that are tolerant of Damage and those that are sensitive to Damage;
- it must be reproducible;
- it should be objective, i.e. independent of any choice the analyst may make;
- it should be general, i.e. applicable to all systems.

The Author maintains that the usefulness of any candidate for a measure of Vulnerability can be assessed against these requirements. Furthermore, according to the Author the measure must be probability-based, because deterministic measures fail to capture the reduction in reliability of a damaged structure, which is considered one essential feature of Damage tolerance.

Vulnerability is quantitatively defined as the ratio between the probabilities of failure of a system in a damaged state and in the pristine state:

$$V=V(r_d,S)=P(r_d,S)/P(r_0,S) \qquad (3.19)$$

where
$P(r,S)$ denotes the probability of failure of the system in a state r for a prospective loading S, r_0 denotes a pristine system state, and r_d denotes a particular damaged state.
Then another definition is proposed in which a set of ordinary undamaged states R_0 and a Damage spectrum R_d are considered:

$$V=P(R_d,S)/P(R_0,S) \qquad (3.20)$$

Damage tolerance is defined as the mathematical reciprocal of Vulnerability:

$$T_d=P(R_0,S)/P(R_d,S) \qquad (3.21)$$

The paper states that research is necessary to develop practical methods for the calculation of Damage tolerance, and that quantitative definition of Damage tolerance makes it possible to specify a minimum allowable value in a code and to set a target value in design for particular classes of systems, but does not provide criteria to decide these values.

The paper includes three simple examples in which Vulnerability and Damage tolerance are calculated. The first two proposed examples refer to a system composed by a finite number of components, and the failure probability of each system component is given.
The third example refers to a structural problem, in which a girder is supported at three points; only one “damaging loading” is considered (the differential settlement of the supports) and only one damaged state is considered (expressed by the stress in the center support). It is given the probabilistic characterization (type of distribution, mean value and variance) of the yield point of the center support, of the stress produced in the support by the ordinary loads, and of the stress produced in the support by the differential settlement; the probabilities needed to apply the definition (3.19) are calculated from these three random variables.

Observations
The work is not targeted specifically to Progressive Collapse, but its concepts can be applied to it. The paper gives a list of qualities that are desirable in a measure of Vulnerability, and stresses the necessity to use a probability-based approach.
The proposed quantitative definition of Damage tolerance is similar to the second Robustness indicator proposed by Maes et al. (section 3.2.1.3).
The proposed definitions are valid for a “spectrum” (or “set”, or “random field”) of loadings, ordinary (undamaged) states and damaged states. The first two proposed examples are easily calculated because the failure probability of the components is given. The third example is one-dimensional, i.e. only one loading variable, one ordinary state and one damaged state are considered. It would be useful to have an example in which the definitions are applied to a multi-dimensional case.

3.3 Summary and comments

Section 3.1 clarifies some aspects about the terms “robust” and “Robustness”, because they are widely used in literature with some ambiguity. Moreover, section 3.2.1.1 describes a paper in which the concept of Robustness is further discussed.

Section 3.2 presented ten ideas that have been proposed for indexes to quantify Progressive Collapse propensity of structure, or that could be applied for this purpose. Several other ideas can be found in literature.

These ideas should be useful to decide if a given structure is safe enough against Progressive Collapse, or if mitigation methods need to be applied. Currently, no building code has adopted any of these methodologies yet; as shown in section 2.2, most building code prescribe conventional verifications to assess if a structure is to be deemed acceptable.
This fact could probably be explained with some considerations:

- some of these ideas are just theoretical constructions and it is not clear how (or even if) they can be actually implemented.
- In some cases only a very general criterion (or no criterion at all) is given to estimate an admissible value of the index and decide if the studied structure is to be considered acceptable. This is very important, because without an admissible value the calculated indexes could just be used to rank the safety of different structures, at the most. For example: according to my index, structure 1 is more vulnerable (or less robust, less resistant...) than structure 2; thus I know that structure 2 is safer than structure 1, but I don't know if it is “safe enough”.
- Most of these ideas require modeling the behavior of a structure, but it is unknown how detailed the models must be to correctly represent the analyzed structure.
- Some ideas follow a deterministic approach, thus the calculated results might not be actually

representative of physical reality.

- In some proposed ideas the calculated parameters refer to just one traumatic event scenario or initial Damage scenario, or to the one scenario that produces the most undesirable consequences. In reality, a traumatic event/initial Damage can happen virtually everywhere in the structure, and with different intensity/extension; by considering only some of these cases, the calculated results might not be actually representative.
- In particular, some ideas quantify an index of Progressive Collapse propensity on the basis of the most undesirable consequence scenario (typically, when the definition requests to consider only the initial Damage scenario that causes a “max” or “min” consequences). This type of indexes do not provide information on how many critical components or areas of the structure exist. As an extreme example, let's consider two structures: in the first one every initial Damage scenario has very small consequences, except one scenario that leads to total Collapse; in the second one every Damage scenario leads to total Collapse. If only the most undesirable consequence scenario is considered, both structures would have the same value of the index.

Section 3.2.4 describes a paper that lists some qualities that are desirable in a quantitative measure of Vulnerability (“*it should be a decision-making tool for design or redesign; it should be calculable; it should be valid*”).

The methodologies that the present work proposes (chapter 5) are based on the considerations here summarized.

Chapter 4 - Incorporating Progressive Collapse in a Risk framework

In this chapter, section 4.1 introduces the concept of Risk.
Section 4.2 describes the probabilistic Risk management framework, developed by the University of Braunschweig, whose concepts are used in this work. Some explanations about the used nomenclature are included.
Section 4.3 describes and analyzes several facts that make it difficult to incorporate Progressive Collapse in a Risk framework.
Section 4.4 summarizes the chapter.

4.1 The concept of Risk

Several different definitions of the term "Risk" exist. As an example, one definition is the following: "*Risk is the potential that an event will lead to an undesirable outcome*".

Every definition shares three elements:
- an event (or a series of events);
- the system affected by the event;
- the consequences of the event on the system.

The concept of Risk involves probability, since the event, or the behavior of the system, or both are not deterministic. In other words, there is never absolute certainty that given consequences will happen (or, conversely, that they will not happen).

The concept of Risk is useful in order to minimize the possibility of unwanted consequences using limited available resources. In order to do it, Risk must be quantified. Thus, numerous quantitative definitions of Risk have been formulated, like the following ones, quoted from [39]:

1. *Risk = hazard x vulnerability x exposure*
2. *Risk = hazard x vulnerability*
3. *Risk = probability x consequences*
4. *Risk = probability x loss*
5. *Risk = probability x damage*

where "hazard", "vulnerability", "exposure", "probability", "consequences" and "damage" are defined and quantified in several ways.

Nowadays almost every type of Hazard is studied by the technical community within a probabilistic Risk framework, because this approach proved to be effective in reducing Losses. Yet, Progressive Collapse is still not studied this way. It is still unknown if it is even possible to do it. The present work is an attempt to understand if, and up to which extent, a Risk framework can be applied to Progressive Collapse.

4.2 The Probabilistic Risk Management Framework

The following excerpts are taken from the paper "*The Probabilistic Risk Management Chain - General Concept and Definitions*", by Pliefke et al. [38]. They are included to explain several concepts of Risk management that will be used in the rest of this work. The used nomenclature will be adopted in the present work, with some modifications to avoid possible confusion.

"RISK MANAGEMENT FRAMEWORK
The general Risk Management *framework consist of three major steps that are performed chronologically. The* Risk Management *process itself consists of three dependent parts which are* Risk Identification*,* Risk Assessment *and* Risk Treatment*.*

After the termination of the three major steps the Risk Review *step is induced. The primary purpose of this step is to constantly include all new information, knowledge and experience about the* Risk *and to indicate its evolution within the process. Accompanying all these steps the* Risk Monitoring *procedure captures the exchange of information of all persons actively or passively involved or participating in the* Risk Management *process. As* Risk Identification *constitutes simply a prerequisite for performing the Risk Management process. The most important tasks can clearly be classified in the* Risk Assessment *and the* Risk Treatment *procedure, which will be characterized subsequently."*

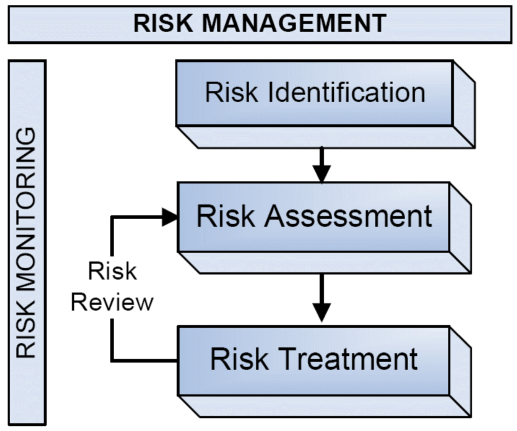

Figure 4.1: The concept of Risk Management and first order subdivision. (Source: [38]).

"Risk Identification
The prerequisite for performing the Risk Identification *phase and therefore to induce the initialization of the* Risk Management *chain is the condition of being aware of a dangerous situation. If this is met first of all the boundaries of the model domain have to be circumscribed by defining the* System *under analysis. Next all sources of* Hazards *that are able to endanger the functionality of the* System *have to be identified. Thus, the* Risk Identification *leads to an answer of the question "what can happen and where???" Eventually, it is proceeded with the* Risk Assessment *phase."*

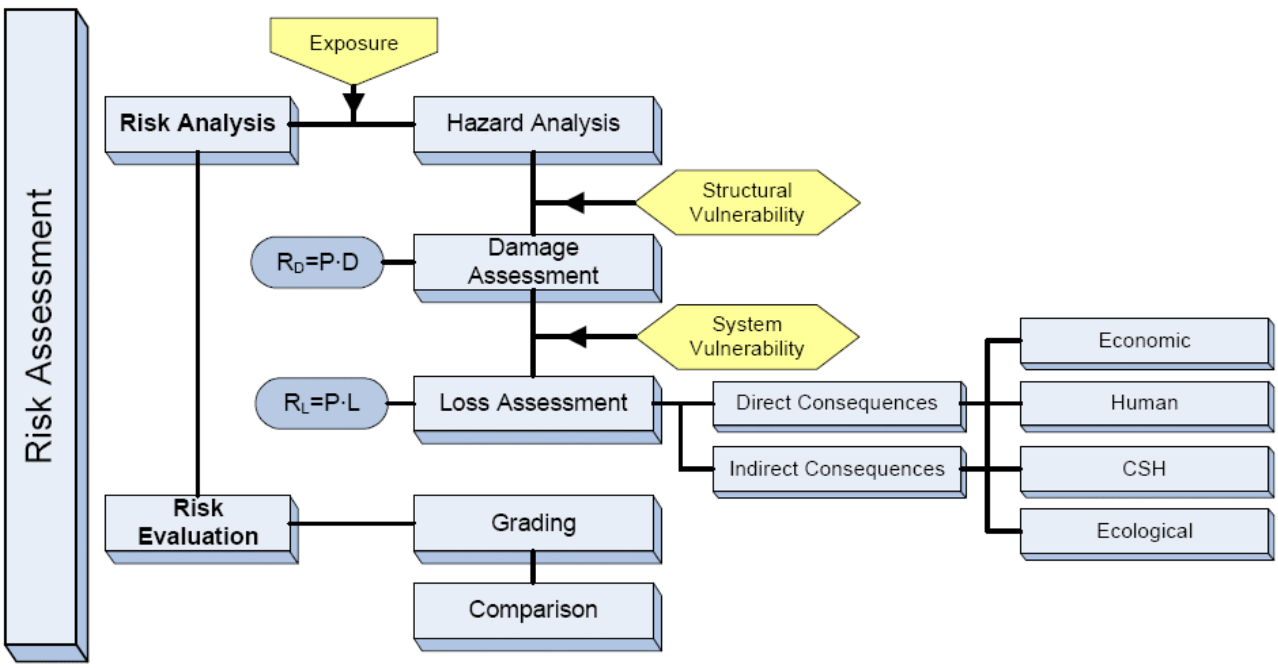

Figure 4.2: The Risk Assessment phase. (Source: [38]).

“Risk Assessment
Given the precondition of having identified the dangerous situation, the Risk Assessment *phase represents the first major step of the* Risk Management *framework. The* Risk Assessment *itself consists of two important procedures that are described more in detail subsequently.”*

“Risk Analysis
The Risk Analysis *procedure is the introductory part of the* Risk Assessment *phase, whose major objective lies in the quantification of the* Risk, *most desirably in monetary units per time unit (e.g. $/year). In order to reach this objective, after having defined the* System *under consideration, a* Hazard Analysis *is being performed where the intensity and frequency of each identified* Hazard *type is estimated. Once the* Hazard *data are quantified, the structural behavior of all the exposed items of the* System, *denoted as* Elements at Risk (EaR), *has to be predicted depending on the* Hazard *intensity.”*
“It should be mentioned that the Structural Damage *captures only the material harm and may be expressed by a larger variety of measures, e.g. water height, crack width or displacement drift. It is not expressed in monetary values. The relation between the* Hazard *intensity and the resulting* Damage *is called* Structural Vulnerability. *Thus, the* Structural Vulnerability *is an indication for the degree of susceptibility of an* EaR *towards the impact of the* Hazard.
The Consequences *that might go in line with a given* Damage *of the* System *have to be analyzed. It is distinguished between* Direct Consequences, *that occur simultaneously to the time the disaster takes place, and* Indirect Consequences, *that occur with a time shift as a result of the* Direct Consequences. *Furthermore each consequence class can be subdivided into tangible or economic consequences, that are directly measurable in monetary terms and intangible consequences, where it is not possible to assign a monetary value in a direct way, e.g. injuries and fatalities, pollution of environment etc.* Indirect Consequences *are to be classified into economic, humanitarian, ecological and CSH (cultural, social, historical) consequences.*
After all possible consequences have been determined, Loss *appraises and eventually accumulates all* Direct *and* Indirect Consequences *at the time the disaster takes place. Thus, the* Indirect Consequences *that occur later in time have to be discounted with a properly defined discount rate that is specific for each consequence class. In this context* System Vulnerability *is an* EaR *specific characteristic, that indicates the total potential of a* Hazard *of a given intensity has on the* EaR.
Thus, System Vulnerability *assigns a* Loss *value to each given* Damage *state of an* EaR *by taking the value of the* EaR *itself as well as its designated functionality within the* System *into account.*
To conclude the Risk Analysis *phase* Risk *can be expressed in two distinctive ways. Firstly,* Risk *can be calculated by taking the product of the annual probability of occurrence for a* Damage *multiplied by the* Damage *itself.*

Structural Risk = Probability x Damage [Damage measure / year] (4.1)

Consequently, the Structural Risk *is of importance primarily to civil engineers in the attempt to predict the behavior and the* Response *of a structure or structural elements. The second way to express the* Risk *is to take the product of the annual probability of occurrence of the* Loss *and the* Loss *itself.*

Total Risk = Probability x Loss [Loss unit / year] (4.2)

It is being referred to as Total Risk. *Thus, by this equation the* Risk *is quantified in a more extensive way as it takes all the possible consequences of the damaged* System *into account.”*

“Risk Treatment
After all the Risks *to the predefined* System *have been analyzed and evaluated the last procedure of*

the Risk Management *framework, the* Risk Treatment *phase, begins to operate. This phase is assigned to the task to create a rational basis for judging on the different* Risks *to the predefined* System *by conducting cost benefit analysis as well as by applying methods from mathematical* Decision *theory. Based on these tools, a* Decision *whether to accept, to transfer, to reject or to reduce a given* Risk *can be derived. In the latter case* Risk Mitigation *initiatives are implemented."*
"When judging on Risk *in the* Risk Treatment *phase, a critical issue might be seen in the broad variety of different individual* Risk *perceptions existing in a society. Whereas one particular individual or group of individuals might have preferences to control one particular* Risk*, another group of individuals might favor the reduction of a distinctive* Risk *to a subjective acceptable level. As there are only limited resources available for* Risk Mitigation *in a society, the public* Risk *reduction interventions cannot focus on one particular group of individuals and manage the* Risk *in line with their interests. Instead, the main objective should rather be to achieve the maximum potential benefit for society as a whole."*
"If the Risk *is to be mitigated,* Decision *makers are given several opportunities to implement a* Risk *reduction project. Firstly, pre-disaster interventions such as* Prevention *and* Preparedness *are available. By definition* Prevention *includes technical measures as well as structural reinforcement projects that are to be performed with an accurate time horizon before the disaster takes place. Typical examples are dykes against floods or dampers against dynamic actions.* Preparedness *contains all social activities, e.g. evacuation plans and emergency training, that are necessary to limit harm shortly before the disaster takes place. Secondly, post-disaster strategies can be followed to reduce the Risk. Among these* Response *covers all activities that are taken immediately after the disaster, such as the organization of help and shelter for the injured and harmed as well as communication between the different emergency forces.* Recovery *subsumes all the activities that need to be taken until the pre-disaster status of the system is reached again. Obviously also a combination of the mentioned possibilities can be used to mitigate the* Risk*."*

Some definitions given in [38] include:
*"**System**:*
The object of investigation for which all sources of Hazard are identified and Risk Analysis is being performed. The System can be composed by a single building or infrastructure element, a suburb of a city, a whole urban region or even an entire country."
*"**Hazard**:*
A potentially adverse physical event, phenomenon or human activity that may cause harm to the predefined System. Harm can include injury or Loss of life, property Damage, cultural, social, historical and economic disruption or environmental degradation."
*"**Element-at-Risk (EaR)**:*
A single or a group of persons or objects within the predefined System that are susceptible and exposed to the impact of a Hazard. In order to guarantee a complete coverage, all Element at Risk collectively should compose the entire System that is being investigated. This will be referred to as the 'principle of completeness'."
*"**Structural Vulnerability: (for each EaR and Hazard intensity)***
Is a specific characteristic of an Element at Risk that indicates the susceptibility towards the impact of a Hazard. Thus, Structural Vulnerability links the Hazard intensity to the Damage of an Element at Risk."
*"**Damage: (for each EaR and Hazard intensity)***
Describes the physical, biological or chemical effect on an Element at Risk caused by the impact of a Hazard of a given intensity. Damage captures the material harm and is not expressed in monetary terms."
*"**System Vulnerability: (for each EaR and Hazard intensity)***
Is a specific characteristic of an Element at Risk, that indicates the total potential of a Hazard of a

given intensity. Thus, System Vulnerability assigns a Loss value to each given Damage state of an Element at Risk. It is best described by a function that evaluates the Consequences of a certain Damage state by taking into account the value of the Element at Risk itself as well as its designated functionality within the System."

*"**Risk Management:***

Risk Management is defined as the systematic application of management policies, procedures and practices to the tasks of identifying, assessing, treating, communicating, reviewing and monitoring Risk."

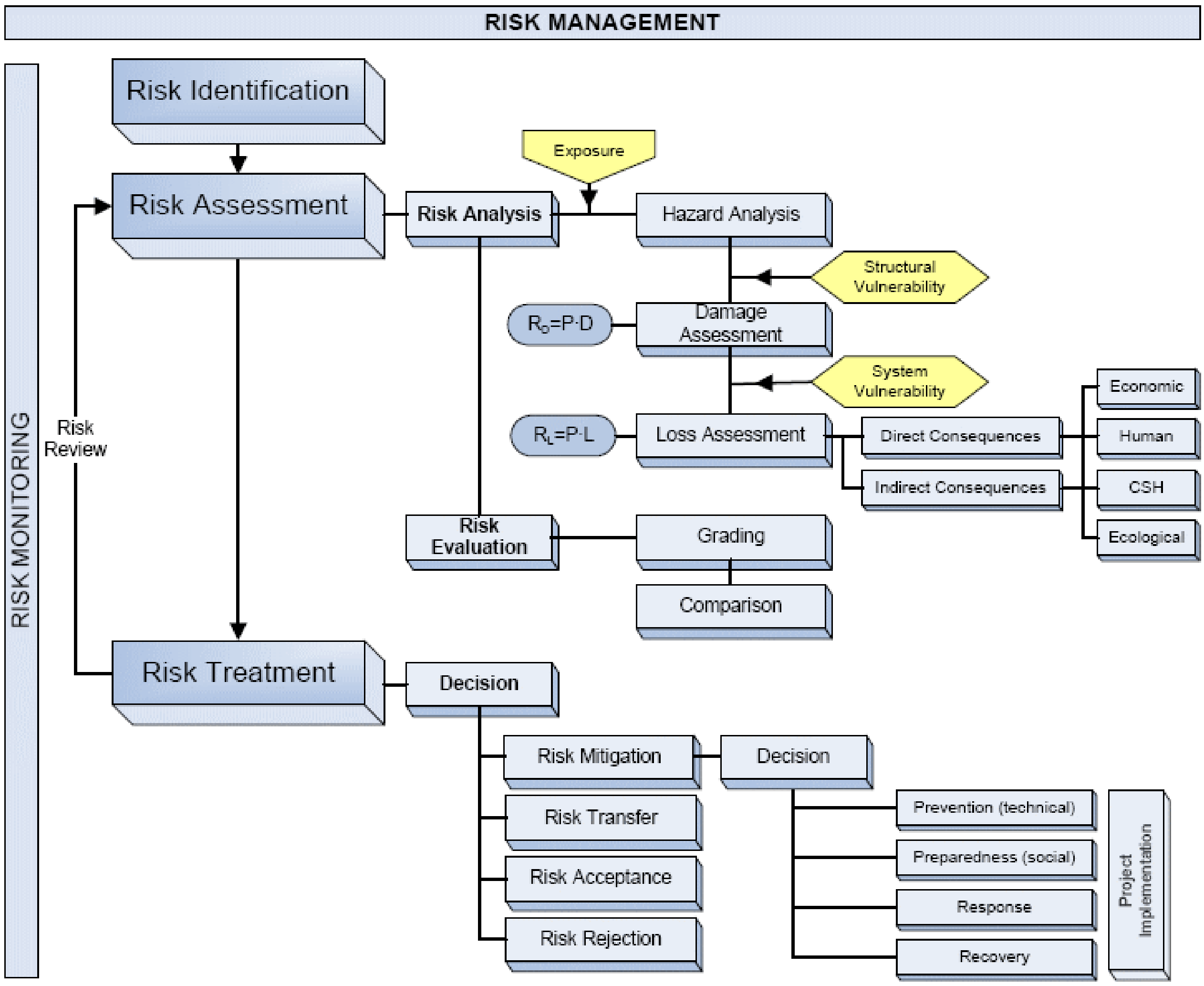

Figure 4.3: Overview of the whole Risk Management process. (Source: [38]).

4.2.1 About the nomenclature

Chapter 3 describes several ideas that "*have been proposed for parameters to quantify the propensity to Progressive Collapse of structures*". Most of these ideas link an *event* (mostly the occurrence of a localized Damage, but in some cases the event that causes the localized Damage) to some of its *direct consequences* (like for example: reduction of the ultimate load, final spread of the Damage, reduction of the determinant of the stiffness matrix, and so on). One exception is the approach described in section 3.2.1.4, in which *indirect consequences* (monetary Losses) are considered. This basic idea is illustrated in figure 4.4, top row.

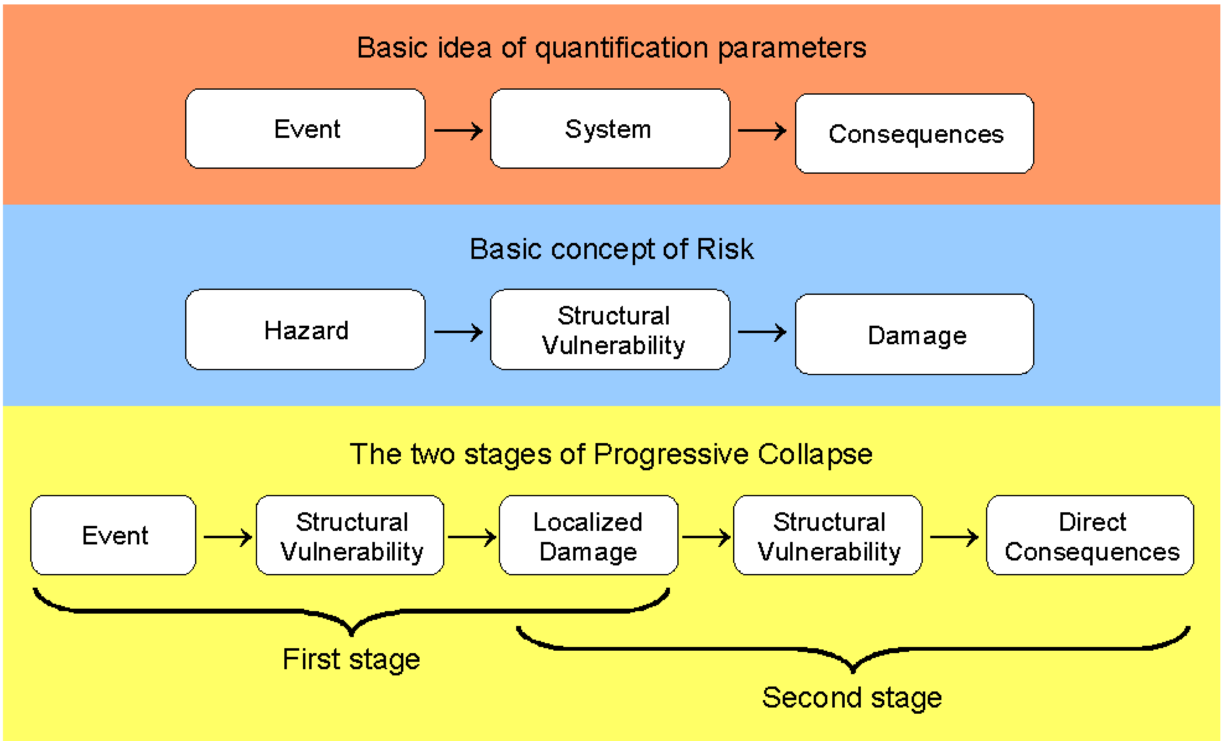

Figure 4.4: Comparison between the basic idea of the quantification parameters (top row), the nomenclature used by Pliefke et al. [38] (middle row) and the distinction of the two stages of Progressive Collapse (bottom row).

These ideas share a similarity with the concept of *Structural Vulnerability*, as defined in [38], whose definition is reported here again:
*“**Structural Vulnerability: (for each EaR and Hazard intensity)***
Is a specific characteristic of an Element at Risk that indicates the susceptibility towards the impact of a Hazard. Thus, Structural Vulnerability links the Hazard intensity to the Damage of an Element at Risk.”
This concept is illustrated in figure 4.4, middle row.

The similarity is that they both relate an *event* (which in [38] is called *Hazard*) to some of its *consequences* (called *Damage* in [38]). Thus, some of the parameters described in chapter 3 would comply with the definition of *Structural Vulnerability* given in [38] (namely, those that follow a probabilistic approach).

Particular attention must be given to the nomenclature.
In [38]:
- the term *Hazard* is defined as *“a potentially adverse physical event, phenomenon or human activity that may cause harm to the predefined System”*,
- and the term *Damage “describes the physical, biological or chemical effect on an Element at Risk caused by the impact of a Hazard of a given intensity.*”

Now, in the context of Progressive Collapse, two stages can be identified:
- a *traumatic event* causes a *localized Damage*;
- the *localized Damage* causes some *direct consequences*.

Confusion can ensue because in each stage there are an *event*, some *consequences* and a relationship that links the two. In particular, *localized Damage* is both *consequence* (in the first stage) and *event*

(in the second stage).
Furthermore, both the relationship that links *traumatic event* and *localized Damage*, and the one that links *localized Damage* and the *direct consequences*, comply with the definition of *Structural Vulnerability.*
These concepts are illustrated in figure 4.4, bottom row.

To avoid confusion, in this work the following nomenclature is used:

- ***Hazard* is any event that can cause a localized Damage in the structure;**
- **the initial *localized Damage* (or, for short, *local Damage)* is the direct effect of the Hazard on the structure;**
- **the term *consequences* denotes the direct consequences of the localized Damage;**
- **the expression *Structural Vulnerability* denotes a function that links a localized Damage and its consequences.**

Furthermore, the term "Hazard" will also be used to denote an entire hazardous phenomenon, when the context of the sentence does not generate ambiguity (for example, "*Earthquake, flood and Progressive Collapse are Hazards that can produce consistent monetary and social Losses*").

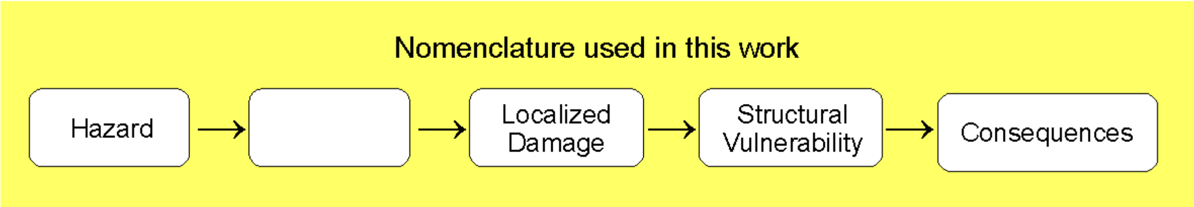

Figure 4.5: Scheme of the nomenclature used in this work. The step between Hazard *and* localized Damage *does not have a specific denomination. The expression "*Structural Vulnerability*" links the* localized Damage *and the* consequences*, exclusively.*

4.3 Risk and Progressive Collapse

As stated in section 4.1, "*Nowadays almost every type of Hazard is studied by the technical community within a probabilistic Risk framework, because this approach proved to be effective in reducing Losses. Yet, Progressive Collapse is still not studied this way. It is still unknown if it is even possible to do it. The present work is an attempt to understand if, and to which extent, a Risk framework can be applied to Progressive Collapse*." These statements deserve to be elaborated on.

4.3.1 Three types of approach

Section 4.1 lists the three basic elements of every definition of Risk:
"- *an event (or a series of events);*
- *the system affected by the event;*
- *the consequences of the event on the system*".

Furthermore, "*the concept of Risk involves probability, since the event, or the behavior of the system, or both are not deterministic*."
In most cases, including in Progressive Collapse, both the event and the system's behavior are non-deterministic. Thus, in order to fully apply the Risk framework, they both need to be probabilistically characterized.
The event is typically characterized by a probability distribution and its parameters, while the system's behavior generally depends on multiple random parameters (such for example the resistance of the materials, their modulus of elasticity, and so on), which in turn can be characterized by probability distributions and their parameters.

If the event and/or the system are not adequately characterized, then it is not possible to calculate a "real" Risk. There are, however, other ways to link event and consequences:

- if the system is adequately characterized but the characterization of the event is missing, then a conventional representation of the event can be used. In this case, the calculated consequences do not fully represent the reality;
- if both the characterization of the event and of the system are missing, then the consequences of the event can be evaluated with a fully "pragmatic" approach. The calculated consequences will be even less representative of reality than the previous described approach.

These three ways to link event and consequences can be summarized as:

(a) both event and system are probabilistically characterized;
(b) the system is probabilistically characterized, the event is conventional;
(c) the event is conventional and the system is deterministic.

Each one of the ideas for parameters to quantify Progressive Collapse propensity listed in chapter 3 falls into one of the three categories (a), (b) or (c), as illustrated in table 4.1.

In particular, table 4.1 summarizes the analyzed ideas and their characteristics:

- the first column identifies the Author of the idea;
- the second column reports the section of this work in which the idea is described;
- the third column identifies which of the above defined categories the idea belongs to;
- the fourth column specifies if the considered event is a Hazard or the occurrence of a localized Damage;
- the fifth column specifies if a probabilistic characterization of the event is considered, or if the event is considered deterministic;
- the sixth column specifies if a probabilistic characterization of the system's behavior is considered, or if the system's behavior is assumed as deterministic;
- the seventh column specifies if the considered consequences are direct or indirect (Losses).

It can be observed that the majority of the listed ideas consider a localized Damage as the event, follow a fully deterministic approach and consider the direct consequences. The cells highlighted by the gray background in table 4.1 are the exceptions.
Out of 10 ideas, 2 are fully probabilistic (category (a)), 1 falls in category (b) and 7 fall in category (c).

			Event		System	Consequences
ID	**Section**	**Category**	**Hazard vs Localized Damage**	**Probabilistic vs Deterministic**	**Probabilistic vs Deterministic**	**Direct Consequences vs Losses**
Maes 1	3.2.1.2	(c)	Damage	Deterministic	Deterministic	Direct
Maes 2	3.2.1.3	(b)	Damage	Deterministic	Probabilistic	Direct
Maes 3	3.2.1.4	(a)	Hazard	Probabilistic	Probabilistic	Losses
Starossek 1	3.2.2.1	(c)	Damage	Deterministic	Deterministic	Direct
Starossek 2	3.2.2.2	(c)	Damage	Deterministic	Deterministic	Direct
Starossek 3	3.2.2.3	(c)	Damage	Deterministic	Deterministic	Direct
Starossek 4	3.2.2.4	(c)	Hazard	Deterministic	Deterministic	Direct
Starossek 5	3.2.2.5	(c)	Damage	Deterministic	Deterministic	Direct
Giuliani	3.2.3	(c)	Damage	Deterministic	Deterministic	Direct
Lind	3.2.4	(a)	Damage	Probabilistic	Probabilistic	Direct

Table 4.1: Summary of the ideas for parameters to quantify Progressive Collapse propensity listed in chapter 3.

4.3.2 Fully probabilistic vs deterministic approaches

What are the advantages of using a fully probabilistic (Risk) approach?
As already stated, this approach gives results that are "closer to reality" than the other ones. Furthermore, it gives more rigorous criteria to decide if a system is acceptable or not.

Although a completely objective acceptability criterion does not exist, one of the most effective ways to decide if a system is safe enough against a given Hazard is to express the Total Risk of that Hazard, i.e. the potential Losses due to that Hazard.
In very simple terms, decision makers (such as politicians, insurers, owners) establish admissible limit values of the Total Risk. By comparing the calculated Total Risk of a system and the admissible value(s), it can be assessed if the system is acceptable or not.
In reality, the safety verifications of modern building codes follow a probabilistic or semi-probabilistic approach, but they are not carried out by explicitly calculating a Risk. Instead, it is implicitly assumed that, if a structure is designed and built according to such codes, the resulting Risk will be lower than an established acceptability limit.

Most of the ideas found in literature for parameters to quantify Progressive Collapse propensity do not have a similarly effective method to establish an acceptability limit.
Without such limit, structures can only be compared and ranked, at the most. For example, one can analyze several different structural configurations of the same building and calculate a Progressive Collapse propensity parameter for each configuration. This will make it possible to answer the question: "*which structural configuration should be preferred?*". The answer, of course, is: "*the configuration whose calculated parameter has the most favorable value*".
But it will not be possible to answer the question "*is this structural configuration sufficiently safe?*" if there is not an absolute reference to compare the calculated parameter to.

4.3.3 Probabilistic characterization of the Hazard

A Risk framework has never been fully applied to Progressive Collapse. This is probably because of two main reasons:

- a Risk framework requires the probabilistic characterization of the considered Hazard;
- applying a probabilistic approach requires a big computational effort.

The first of these issues is discussed in this section.

The occurrence of Progressive Collapse is relatively rare, thus data about it are scarce.
Some Authors consider Progressive Collapse a "low probability/high consequences" Hazard, i.e. an Hazard whose consequences can be severe, but whose probability of occurrence is so low that typical probabilistic methods cannot be applied to it. For example, Bontempi [8] states that[1] "*safety in simple situations is estimated through qualitative deterministic analyses which, as the complexity of the structural problem increases, are replaced by more refined analyses based on probabilistic considerations. As complexity further increases this trend is reversed, going back to deterministic approaches, i.e. pragmatic analyses are considered, which are based on risk scenarios based on expert judgment, which transcend the mere statistical descriptions*".
It must be noticed that most building regulations follow this "pragmatic" approach. For example, the Alternate Load Path analysis is threat independent (the location and extension of the initial Damage is chosen conventionally, rather than following probabilistic criteria) and the structure's behavior is modeled as deterministic.
In the surveyed ideas (table 4.1), only 2 out of 10 follow a fully probabilistic approach, and in the presented examples very specific Hazards are considered and characterized (the event that a forklift impacts on a wall and the event of differential settlement of beam supports). It must be remembered

1 Translated from italian.

that, in the general case, many types of event can provoke the localized Damage that can prompt a Progressive Collapse, as highlighted in sections 1.3 and 2.2.1.1.

Now, one question arises: are the available data really insufficient to adequately characterize the events that can prompt a Progressive Collapse? This is still unknown. One thing is objectively sure: there is a consistent amount of available information that has not been collected yet.
It is worth to remember (see section 2.2.1.1) that:

- Allen and Schriever [4] found that 15% to 20% of surveyed Collapses had to be considered progressive;
- the mean occurrence rate of some exceptional Hazards (vehicle impact, gas explosion, bomb explosion) has been deemed not negligible (in the order of 10^{-6} to 10^{-4}/yr; Leyendecker and Burnett [23]);
- most of the available statistics were carried out in the 1970s (following the resonance of the Ronan Point Collapse) and typically refer to events of that decade and of the previous one;
- in their 1976 work [23], Leyendecker and Burnett stated that "*a number of assumptions have been necessary in order to analyze the data presented in this report. These are related primarily to the lack of detailed descriptions of damage accompanying the various abnormal loading events and the lack of detailed descriptions of the buildings involved in the incidents*" and advise that "*the statistical reporting of abnormal loading events needs to be considerably improved in order to obtain load data and damage data in particular types of building construction*".

Thus, our knowledge would undoubtedly increase by performing extensive surveys and by creating databases to be updated according to specific criteria after each new event.

4.3.4 Reliability methods

In order to calculate a Risk, reliability methods must be used. By definition, these methods estimate the probability that a system fulfills its required performance during a specified period of time under stated conditions.
This is typically done numerically, by means of models that simulate the behavior of the system. Most reliability methods require a consistent number of model runs, up to the order of magnitude 10^6 and more.

In Progressive Collapse, detailed non-linear models are often required. Applying typical reliability methods to these models would generally require computation times in the order of years with the current technologies, which would be deemed unacceptable.
Thus, in the methodologies presented in chapter 5 of the present work the First Order Reliability Method (FORM) is used, which is more approximated than other methods but faster. The detailed description of the used FORM procedures is reported in section 5.3.

4.4 Summary

In this chapter, the concept of Risk is introduced in section 4.1.
Section 4.2 describes the probabilistic Risk management framework and points out some aspects of the nomenclature used in this work.
Section 4.3 presents and elaborates on aspects that make it difficult to incorporate Progressive Collapse in a Risk framework.

This information will be the basis for the methodologies presented in chapter 5.

Chapter 5 - The proposed methodologies

In this chapter, two methodologies to quantify Progressive Collapse propensity of frame structures are presented.
Section 5.1 lists the motivations to the development of these methodologies and the targets they aim to.
Section 5.2 explains the basic ideas of the methodologies.
Section 5.3 explains the details of how the methodologies can actually be implemented.
Section 5.4 summarizes the chapter and elaborates on it.
In the following chapter, the first proposed methodology is applied to several examples.

5.1 Motivations and target

From the information given in the previous chapters, several considerations can be obtained.

- Progressive Collapse is a phenomenon that can produce consistent Losses.
- Its occurrence is relatively rare, but it has been deemed not negligible.
- Many building codes require measures to contrast Progressive Collapse, but these measures generally follow a conventional, “pragmatic” approach and it is unknown how effective they really are.

- Several ideas for parameters to quantify the propensity to Progressive Collapse have been proposed in literature.
- No one of these ideas has been incorporated in a building code, yet.
- The majority of the proposed ideas follow a deterministic approach.
- Those that follow a probabilistic approach usually refer to a single predetermined Hazard and one single initial Damage scenario, and might require a considerable amount of work and time to be applied.
- With some ideas, only theoretical reasonings are given, and it is unclear how you actually calculate the parameter.
- In general, there is not an objective criterion to decide from the calculated parameter if a structure is to be considered safe enough against Progressive Collapse.

This led to the following “wish list”, i.e. a list of qualities that are desirable in a quantification methodology:

(a) It should be reliable.
(b) It should be usable in common practice.
(c) It should be a decision tool.
(d) It should consider the entire studied structure.

More in detail:

(a) The methodology should be reliable. *Reliability* is generally defined as the ability of a system to perform its required functions. In our context the system is the quantification methodology and the required function is the quantification of Progressive Collapse propensity of a given structure. Our methodology should give us results that really represent the physical reality. Since no model will ever replicate reality exactly, having a reliable methodology means that we always should be aware of how distant from physical reality our results can be.
(b) The methodology should be *practical*, i.e. it should be actually usable. In particular, it

should not contain steps that are theoretically valid but not actually feasible and should not require excessive effort to be implemented and used. If it was possible to devise a methodology that provides high quality information but requires excessive effort (like months or years of computation time), then none would use it in practice and it would be like having no methodology.

(c) This is its ultimate target: the methodology should be a decision tool ("*is the analyzed structure acceptable or not?*").

(d) The methodology should consider the entire studied structure, because the traumatic event that prompts a Progressive Collapse can occur virtually everywhere, and with different intensities.

Since perfection does not exist, it will be impossible to have all the qualities of the wish list completely fulfilled. Thus, some compromises will be accepted. In particular, it will be considered acceptable to introduce approximations, or to reduce the quality of the obtained information, if the problem becomes too complex or cumbersome. If approximations are introduced, then we would like to have an idea of the maximum error introduced and we would prefer if they are on the "safe side". Of course, every approximation could be reduced in future improvements of the methodology.

In short, the target of the present work can be summarized as:
attempting to understand if, and up to which extent, it is possible to devise a reliable and practical method to quantify the propensity to Progressive Collapse of a given structure, as well as to quantify an acceptable level of this propensity.

5.2 The proposed methodologies

The target of this work, expressed at the end of the previous section, can be reformulated as trying to answer to these two questions:

- *Given a structure, how safe is it against Progressive Collapse?*
- *How much safety do we need?*

This is done by trying to incorporate Progressive Collapse in a probabilistic Risk framework. In terms of Risk, the previous questions can be reformulated as:

- *Given a structure, what is its Risk of Progressive Collapse?*
- *How much Risk can we accept?*

5.2.1 Solving the problem – The basics

The logic of the proposed methodologies originates from the following equation, which is presented by Ellingwood in many of his works (for example in [35]) and that can be traced back to Leyendecker and Burnett (1976) [23]:

$$P(C)=P(C|LD)P(LD|H)P(H) \tag{5.1}$$

where P(C) is the probability of occurrence of a Progressive Collapse;
P(H) is the probability of occurrence of a given Hazard;
P(LD|H) is the probability of having a Local Damage given the Hazard;
P(C|LD) is the probability of having a Collapse given the Local Damage.

Several observations about equation (5.1) can be pointed out.

- Each term of equation (5.1) corresponds to one of the first three mitigation strategies described in section 2.1. Each strategy aims at reducing P(C) by reducing one of the three terms: the Event Control strategy aims at reducing P(H); Specific Load Resistance aims at reducing P(LD|H); Alternate Load Path aims at reducing P(C|LD).

- The equation has never been actually used to calculate a probability of Collapse P(C). Ellingwood uses it mainly to illustrate principles of reliability analysis such as those to obtain the formula (2.3) (i.e. the load combination for the Alternate Load Path analysis of ASCE 7).
- In [35] the same Author provides additional useful information: he reports and elaborates on P(H) for some Hazard types; he states that in an Alternate Load Path approach Local Damage is accepted and thus P(LD|H)≈1; he states that P(C|LD) can be calculated with reliability methods. Furthermore, he states that "*...[the] risk below which society normally does not impose any regulatory guidance, is on the order of 10^{-7}/yr. [...] we may take 10^{-7}/yr as a target value [of P(C)], with the understanding that final decisions regarding acceptable building risk are outside the scope of this document*".
- In this formulation C is a binary random variable, which for example may assume the value 1 if the initial Local Damage extends and 0 if it does not. The final extension of the Damage is not considered. While a sound estimate of P(C) for a given structure would be useful information for decision makers (legislators, building owners, insurers...), a probabilistic description of the Final Damage extension would be better. This can be achieved by defining the random variable C to represent the final extension of the Damage. In this work, two methodologies are presented; one that does not consider the extension of the Damage, and one that does. The extension from the first to the second methodology is conceptually simple, but it requires a much bigger effort.
- The value of P(C) will actually be given by the sum of many contributions, since the Local Damage can happen in many areas of the structure and with many intensity levels.

5.2.2 First methodology

The following assumptions are made:

- Eq. (5.1) is rewritten as

$$P(C)=P(C|LD)P(LD) \tag{5.2}$$

  where P(LD)=P(LD∩H)=P(LD|H)P(H).
- The initial Local Damage LD and the Collapse C are assumed as discrete random variables. Thus, the notation P( ) denotes a probability mass function, i.e. a function that gives the probability that a discrete random variable is exactly equal to some value.
- The random variable C can only assume the two values 0 (when the Damage does not extend) and 1 (when it does).
- The methodology is restricted to frame structures.
- The Local Damage level is quantified as the number of "failed" structural elements, i.e. as the number of elements that are not able to fulfill any of their functions anymore (see figure 5.1). Thus, we can have LD=0 (undamaged structure), LD=1 (one structural element is failed, while the other ones are not), LD=2 (two elements are failed), and so on. This assumption is further discussed in section 8.1.3.
- The statistical description of P(LD) is known for the analyzed structure. This assumption is further discussed in section 5.3.1.3.
- Geometry and materials of the structure are known; they depend on some random variables, whose probability distribution is known.
- The loads are random parameters, of known probability distribution.
- The behavior of the structure can be modeled with sufficient accuracy (section 8.1.2 discusses the meaning of "sufficient accuracy").

It must be noticed that the methodology could be improved, in the future, by changing some of these assumptions. Chapter 8 presents and discusses several of these possible improvements.

To simplify the explanation, let's assume at first that only one Damage scenario is possible for each value of the Local Damage level LD. Under all these hypotheses equation (5.2) can be rewritten as

$$P(C=1)=\sum_{i}^{n} P(C=1|LD=i)P(LD=i)$$

or, with a more concise notation,

$$P(C=1)=\sum_{0}^{n}{}_{i} P(C=1|LD_i)P(LD_i) \tag{5.3}$$

where n is the total number of structural elements in the structure.
Equation (5.3) basically states that the total probability of having a Collapse is the sum of the probabilities for each initial Local Damage level.

If we remove the assumption that only one Local Damage scenario can happen for each initial Local Damage level, the equation becomes

$$P(C=1)=\sum_{0}^{n}{}_{i}\sum_{1}^{t}{}_{k} P(C_{ik}=1)=\sum_{0}^{n}{}_{i}\sum_{1}^{t}{}_{k} P(C=1|LD_{ik})P(LD_{ik}) \tag{5.4}$$

where t is the total number of Local Damage scenarios for the i-th Local Damage level.
Equation (5.4) basically states that the total probability of having a Collapse is the sum of the probabilities of Collapse for each initial Local Damage scenario and for all initial Local Damage levels.
In the used notation the first index refers to the Local Damage level and the second to the Local Damage scenario. For example, $P(LD_{ik})$ is the probability of occurrence of the k-th scenario of the i-th Local Damage level. $P(C_{ik}=1)=P(C=1 \cap LD_{ik})$ is a concise notation for the joint probability of Collapse occurrence and a given Local Damage scenario.

In order to calculate P(C=1) using equation (5.4), three main elements are needed:
- the $P(C=1|LD_{ik})$ terms need to be calculated;
- the $P(LD_{ik})$ terms must be evaluated;
- it must be assessed how many Local Damage scenarios need to be considered.

These aspects are analyzed in section 5.3.

5.2.3 Second methodology

The second methodology differs from the first one in that it considers the final extension of the Damage.

The following assumptions are made:
- The methodology is restricted to frame structures.
- The geometry and the materials of the structure are known; they both depend on some random variables, whose probability distribution is known.
- The loads are random parameters, of known probability distribution.
- The statistical description of the initial Local Damage P(LD) is known for the analyzed type of structure.

- Given an initial Local Damage, its progression in the structure can be modeled with sufficient accuracy.
- For each initial Local Damage scenario, every set of the random variables corresponds to one Final Damage scenario.
- The final extension of the Damage is expressed as the number of failed structural elements; the Final Damage level FD is assumed as a discrete random variable.

Under these assumptions the total probability of having the Final Damage level f is:

$$P(FD=f)=\sum_{i\,0}^{f}\sum_{k\,1}^{t}P(FD_{ik}=f)=\sum_{i\,0}^{f}\sum_{k\,1}^{t}P(FD=f|LD_{ik})P(LD_{ik}) \qquad (5.5)$$

In order to calculate P(FD=f) using equation (5.5), three main elements are needed:
- the $P(FD=f|LD_{ik})$ terms need to be calculated;
- the $P(LD_{ik})$ terms must be evaluated;
- it must be assessed how many Local Damage scenarios need to be considered.

As with the first methodology, these aspects are analyzed in section 5.3.

5.2.4 Observations

With reference to the probabilistic Risk management framework developed by the University of Braunschweig [38] (described in section 4.2) some observations can be made about equation (5.2), which is reproduced here again:

$$P(C)=P(C|LD)P(LD)$$

Since
- the Local Damage LD is *"a potentially adverse physical event, phenomenon or human activity that may cause harm to the predefined System"*, i.e. it complies with the definition of *Hazard* according to [38];
- and the occurrence of a Collapse C *"describes the physical, biological or chemical effect on an Element at Risk caused by the impact of a Hazard of a given intensity"*, i.e. it complies with the definition of the term *Damage*;

then
- the term P(C|LD), which "*links the Hazard intensity to the Damage*", complies with the definition of *Structural Vulnerability*;
- and P(C) complies with the definition of *Structural Risk*.

It is important to remind that in the present work the terms *Hazard* and *Damage* have a different meaning than in [38], as explained in section 4.2.1. In this section [38] is quoted to show the terms that comply with the definitions of *Structural Vulnerability* and *Structural Risk*.

Similarly, in equation (5.5):

$$P(FD=f)=\sum_{i\,0}^{f}\sum_{k\,1}^{t}P(FD=f|LD_{ik})P(LD_{ik})$$

- the terms $P(FD=f|LD_{ik})$ comply with the definition of *Structural Vulnerability*,
- and the term P(FD=f), multiplied by f, complies with the definition of *Structural Risk*.

Thus, calculating P(C=1) or P(FD=f) gives an answer to the question "*Given a structure, what is its*

Risk of Progressive Collapse?”, formulated in section 5.2 as one of the targets of the present work, in terms of Structural Risk (i.e., expressed in terms of direct consequences).

The answer to the second question “*How much Risk can we accept?*” in general can only be decided when it is expressed in terms of Total Risk, i.e. in terms of Losses, and is usually demanded to decision makers.
This decision is case-specific, because both the acceptable Losses and the relation between direct consequences and Losses (i.e., between Structural Risk and Total Risk) generally depend on the specific context. Thus, in this work the relationship between Structural Risk and Total Risk is not studied, and the question “*How much Risk can we accept?*” is not explicitly answered.
In the examples presented in chapter 6 the acceptable “target value” of the Risk suggested by Ellingwood $P(C=1)_{acc}=10^{-7}$/yr is used (as reported in section 5.2.1). Of course, specific cases can lead to different values of the acceptable Structural Risk.

5.3 Implementation of the methodologies

The following sections describe how each element needed to implement of the proposed methodologies can be obtained.

5.3.1 Implementing the first methodology

In order to calculate the Structural Risk $P(C=1)$ by using equation (5.4):
- the Structural Vulnerability terms $P(C=1|LD_{ik})$ need to be calculated;
- the $P(LD_{ik})$ terms must be evaluated;
- it must be assessed how many Local Damage scenarios must be considered.

The following sections deal with these issues.

5.3.1.1 Calculation of the Structural Vulnerability

The defined Structural Vulnerability $P(C=1|LD_{ik})$ is the probability that a Collapse originates after the structure has suffered a Local Damage scenario LD_{ik}.
In the present context, “Collapse” means that at least one connection in the structure is lost or, in other words, a section has lost all its resistance properties, resulting in physical separation. It must be highlighted that a section might lose some of its resistance properties (e.g. bending moment) while retaining others (e.g. axial force), as in the example of figure 5.1.

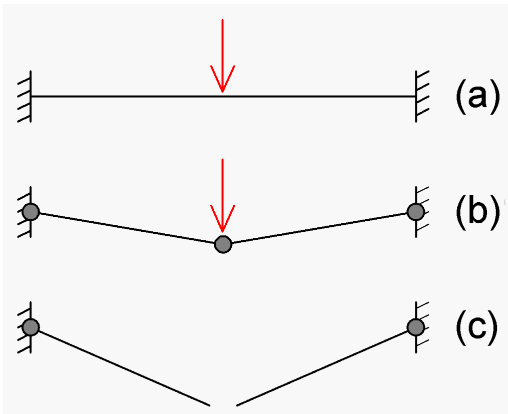

Figure 5.1: In this example, in configuration (b) the beam has lost bending moment resistance in three sections but is still able to bear the load through axial force. In configuration (c) one section becomes detached and the element is considered “failed”.

$P(C=1|LD_{ik})$ can be calculated with reliability methods. By definition, these methods estimate the probability that a system fulfills (or, equivalently, that it does not fulfill) its required performance during a specified period of time under stated conditions. In our case the system is a structure that has suffered a given Local Damage and the required performance is the avoidance of Collapse.

The basic principle of reliability methods is that:
- given a vector **x** of the random variables x_i of the problem (such as system properties, loads, model uncertainties),
- and its joint probability density $f(\mathbf{x})$,

- a *performance function* g(**x**) is defined as an arbitrary function of the random variables vector **x**, which assumes positive values when the required performance is fulfilled by the system ("success" condition) and negative values when the it is not ("failure" condition).

Then the probability of failure P_f is given by

$$P_f = \int_{g(\mathbf{x}) \leq 0} f(\mathbf{x}) d\mathbf{x} \qquad (5.6)$$

In the general case the integral (5.6) does not have an analytical solution, so numerical methods must be used. Numerical solution with simulation methods (such as Monte Carlo integration and its variants) requires a considerable amount of calculations, so the presented methodologies would take unacceptable time to be performed (in the order of years with the current technologies). Thus, in the present work the First Order Reliability Method (FORM) is used, which is approximated but faster. Other, more precise methods might also work. It must also be noticed that section C2.5 of ASCE 7 suggests the use of the FORM to calculate this probability, but it does not elaborate further (as reported in section 2.2.2.2).

In this work, two FORM procedures are used, both devised by Val et al. ([49], [50], [51], [52]). They are hereinafter referred as "*standard FORM procedure*" and "*simplified FORM procedure*"; as the name suggests, the second procedure is more approximated than the first one, but it is much faster to run.

The following sections illustrate the basic principles of the FORM and the two used FORM procedures.

5.3.1.1.1 The First Order Reliability Method (FORM)

With the First Order Reliability Method (FORM) an approximated value of the integral (5.6) is calculated with the methodology herein described.

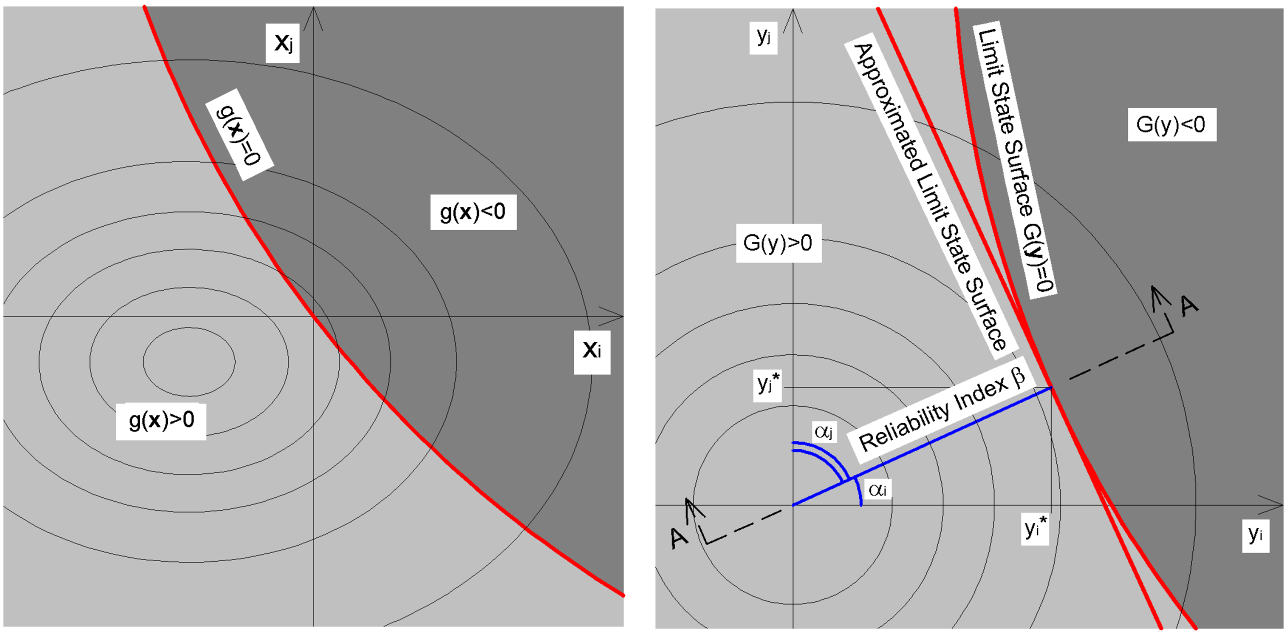

Figure 5.2: Representations of a probability space (left) and of the corresponding standardized Gaussian space (right).

Each random variable x_i of the vector **x** is characterized by a probability distribution and its parameters (such for example the mean value μ_i and the standard deviation σ_i). It is assumed that the type of distribution, as well as the necessary parameters, are known for each variable.

The random variables vector **x** is transformed into a vector of standard Gaussian variables **y,** i.e. a vector in which each entry y_i is a Gaussian random variable with mean value equal to zero and standard deviation equal to one. For example, a generic Gaussian variable x_i becomes the standard Gaussian variable

$$y_i=(x_i-\mu_i)/\sigma_i \qquad (5.7)$$

For non-Gaussian variables, the probability distribution is replaced with a local Gaussian approximation, which is then standardized.

In the standard Gaussian space the performance function g(**x**) becomes the function G(**y**) such that, if $\mathbf{y_0}$ is the standardized vector of $\mathbf{x_0}$, then $G(\mathbf{y_0})=g(\mathbf{x_0})$. The locus of G(**y**)=0 is called *limit state surface* and it is the boundary that separates the success states (in which G(**y**)>0) from the failure states (G(**y**)<0).

The limit state surface is then approximated with a linear expansion at the point **y***, which is the point of the limit state surface closest to the origin of the standardized space.
By approximating the limit state surface with the linear expansion, the integral (5.6) can be calculated as

$$P_f=1-\Phi(\beta)=\Phi(-\beta) \qquad (5.8)$$

where

- Φ is the cumulative distribution function of the standard Gaussian distribution,
- and the parameter $\beta=(\mathbf{y^{*t}}\cdot\mathbf{y^*})^{1/2}$ is called *reliability index* and is defined as the distance between the approximated limit state surface and the origin.

The relationship (5.8) can be best comprised by imagining to "slice" the standardized Gaussian space along the direction of β; in figure 5.2 (right), this is indicated as "section A-A".
Figure 5.3 (left) represents the section, which is assumed to be an approximated[1] representation of the probability density function φ of the standard Gaussian distribution.

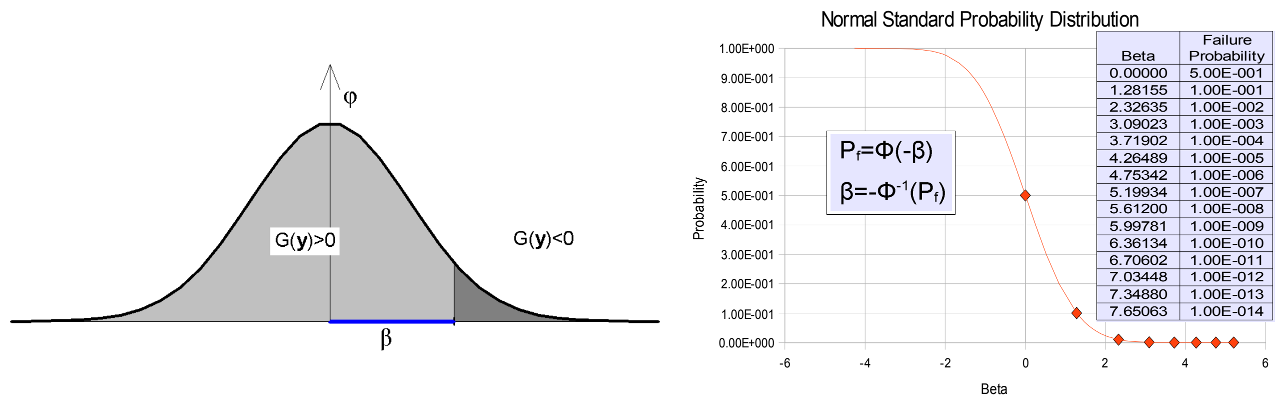

Figure 5.3: (Left) Representation of section A-A from figure 5.2. (Right) Relationship between reliability index β and probability of failure P_f.

Calculating the reliability index β is basically a minimization problem, which can be concisely expressed as

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1 The relationship (5.8) is actually approximated. According to Rackwitz and Fiessler [41] the actual probability of failure P_f corresponding to β lies in the range $1-\Phi(\beta)<P_f<1-\chi_n^2(\beta^2)$, where χ_n^2 is the chi-square distribution with n degrees of freedom, but "*in general, the lower bound is a sufficiently accurate estimate of the exact failure probability*".

$$\beta=\min\{(\mathbf{y}^t\cdot\mathbf{y})^{1/2}\} \text{ given the condition } G(\mathbf{y})=0 \qquad (5.9)$$

The main approximation of the FORM is that the area comprised between the real limit state surface and the approximated one (figure 5.2, right) is included in the "failure" condition, even though in that area $G(\mathbf{y})>0$. Thus, compared to the rigorous solution of equation (5.6), the probability of failure P_f is overestimated.

If $\boldsymbol{\alpha}$ is the unit vector of the direction cosines of vector $\mathbf{y^*}$, then it is

$$\mathbf{y^*}=\beta\boldsymbol{\alpha} \qquad (5.10)$$

The entries of $\boldsymbol{\alpha}$ are called *sensitivity factors*, because they provide an indication of the relative importance of the corresponding random variable in determining P_f. In particular, it has been shown [50] that if the i-th random variable is replaced with its mean value, then the reliability index β increases by a factor of $1/(1-\alpha_i^2)^{0.5}$.

5.3.1.1.2 The standard FORM procedure - Explanation

This section explains how the standard FORM procedure works. The next section presents an example of its application.

The presented procedures are specifically targeted at Progressive Collapse, thus in this context:
- the considered system is a structure;
- the required performance is the avoidance of Collapse;
- "failure" condition is identified with the occurrence of Collapse and "success" with the avoidance of it;
- the calculated probability of failure corresponds to the generic Structural Vulnerability term, i.e. $P_f \equiv P(C=1|LD_{ik})$.

The performance function is defined as

$$g=\xi R-S \qquad (5.11)$$

where
- R is the resistance of the structure;
- S is the total applied load,
- and ξ represents the uncertainty of the structural model.

The basic concept behind the formula (5.11) is quite simple: Collapse occurs when the applied loads are higher than the structure's resistance. An extra term ξ is included to take into account the uncertainty of the used structural model.

The applied load S is generally given by the sum of several contributions, such as permanent + live load. Each contribution can be identified by a random variable, whose probability distribution and parameters is assumed to be known; their characterization can be obtained from several literature sources.

The resistance R is defined as the maximum load that the structure is able to bear without inducing spread of the Damage. It will depend on several random parameters, such as the structure's material properties. The probabilistic characterization of these parameters can be obtained from literature and it is assumed to be known.
In general an analytical description of the resistance R cannot be formulated, thus R can be

considered an unknown function of known random variables. Single values of R, for a given set of values of its random parameters, can be calculated by means of a model.

The model uncertainty ξ is also considered as a random variable of known characteristics.

The performance function (5.11) is a function of several known random variables, but its analytical description is not available because the analytical description of R is missing. As a consequence, the probability of failure P_f can only be calculated numerically.
As stated, calculating P_f is basically corresponds to the minimization problem (5.9). This type of problems is usually solved numerically with gradient methods, such as Hasofer-Lind/Rackwitz-Fiessler [41]. The Authors of the FORM procedures [49] report that the usual numerical gradient methods often fail to converge with this specific problem, because the calculated values of the gradient are too approximated. Thus, the conjugate direction method is used (Brent [9]); compared to the gradient methods it has few convergence problems, although it requires more model runs.

To implement the conjugate direction method, first the m-dimensional vector **x** is defined, in which each entry x_i is a random variable of the problem. Then **x** is transformed into the corresponding vector of standard Gaussian variables **y**.
Then **y** is expressed in polar coordinates, according to the following relations:

$$y_m = r \sin \varphi_1$$
$$y_{m-1} = r \cos \varphi_1 \sin \varphi_2$$
$$\ldots$$
$$y_i = r \cos \varphi_1 \cos \varphi_2 \ldots \cos \varphi_{m-i} \sin \varphi_{m-i+1}$$
$$\ldots$$
$$y_2 = r \cos \varphi_1 \cos \varphi_2 \ldots \cos \varphi_{m-2} \sin \varphi_{m-1}$$
$$y_1 = r \cos \varphi_1 \cos \varphi_2 \ldots \cos \varphi_{m-2} \cos \varphi_{m-1}$$

Now the problem (5.9) can be rewritten as

$$\beta = \min\{r\} \text{ given the condition } G(r, \boldsymbol{\varphi}) = 0 \qquad (5.12)$$

where r is the radius coordinate and $\boldsymbol{\varphi}$ is the (m-1)-dimensional vector of the angle coordinates.
It must be highlighted that, in the used notation, $\boldsymbol{\varphi}$ (bold) represents a vector, while φ_i (normal) represents one entry of the vector. Thus a notation like $\boldsymbol{\varphi_i}$ means "the i-th vector", while φ_i means "the i-th entry of a vector".

To solve the problem (5.12), the performance function g is first calculated in an arbitrary starting point $\mathbf{x_0}$.
The values of the random variables of the total applied load S, as well as the uncertainty of the structural model ξ, are entries of $\mathbf{x_0}$, so they are known.
The value of the resistance R must be calculated by means of a model of the structure, in which the value of the random parameters is given by $\mathbf{x_0}$.

One method to evaluate R is to perform a static nonlinear analysis for increasing vertical loads (a.k.a."pushdown analysis"). The vertical load is applied in small increasing steps; R is taken as the load corresponding to the last step before Collapse condition occurs.
Another, more laborious method to evaluate R is to perform dynamic analyses, in which the damaged part is suddenly removed from the loaded structure. This way, each analysis run can result in Damage spread or not. The runs must be repeated for different values of the load, in order to obtain an estimate of the critical load, i.e. of the minimum load value that results in Damage spread.

Section 8.1.1 further elaborates on the dynamic analysis.

Once the value of the resistance R is obtained, the value of the performance function is calculated with equation (5.11). By definition, the calculated value $g(\mathbf{x_0})$ is equal to $G(r,\boldsymbol{\varphi_0})$.
From this starting point, the critical condition G=0 is found moving along r. This involves changing the value of r while maintaining $\boldsymbol{\varphi_0}$ fixed, calculating the corresponding $\mathbf{x}$ vector and repeating the previously described step.

The minimum of $r(\boldsymbol{\varphi})$ given the condition $G(r,\boldsymbol{\varphi})=0$ is reached by moving along conjugate directions in the m-1 dimensional space of the angles $\boldsymbol{\varphi}$.
By definition, the vectors $\mathbf{d_1}$, $\mathbf{d_2}$, ..., $\mathbf{d_{m-1}}$ constitute a set of conjugate directions with respect to a definite positive matrix $\mathbf{A}$ if they are linearly independent and $\mathbf{d_i}^t\mathbf{A}\mathbf{d_j}=0$ for all $i\neq j$.
Brent [9] shows that the minimum of the quadratic function $f(\mathbf{u})=\mathbf{u}^t\mathbf{A}\mathbf{u}/2+\mathbf{b}^t\mathbf{u}+c$ can be found by m-1 one-dimensional minimizations along these directions, in whatever order.

It is suggested that the initial search directions $\mathbf{d_1}$, $\mathbf{d_2}$, ..., $\mathbf{d_{m-1}}$ are taken as the columns of the identity matrix. The radius r is minimized by moving along these m-1 directions. Then a new set of directions is defined, in which $\mathbf{d_i}=\mathbf{d_{i+1}}$ for i=1 to m-2 and $\mathbf{d_{m-1}}=\boldsymbol{\varphi_{m-1}}-\boldsymbol{\varphi_0}$. One more minimization along the direction $\boldsymbol{\varphi_{m-1}}-\boldsymbol{\varphi_0}$ is then performed; the reached point is chosen as the new $\boldsymbol{\varphi_0}$.
After m-1 repetitions the directions $\mathbf{d_1}, \mathbf{d_2}, ..., \mathbf{d_{m-1}}$ will be conjugate. If the problem is quadratic, the minimum is reached; otherwise, the algorithm can be repeated until a stop criterion is satisfied.

5.3.1.1.3 The standard FORM procedure – Example

This section presents a step by step example of how the algorithm works with two random variables (thus, $\mathbf{y}\in R^2$ and the polar coordinates r,φ_1 are both scalars). In the end, the extension to more variables is explained.

A starting vector of the random variables $\mathbf{x_0}$ is arbitrarily chosen. In the standard Gaussian space it corresponds to vector $\mathbf{y_0}=(y_1^{(0)}, y_2^{(0)})\equiv(r^{(0)}, \varphi_1^{(0)})$. (The upper right index between parentheses denotes the current step of the algorithm).

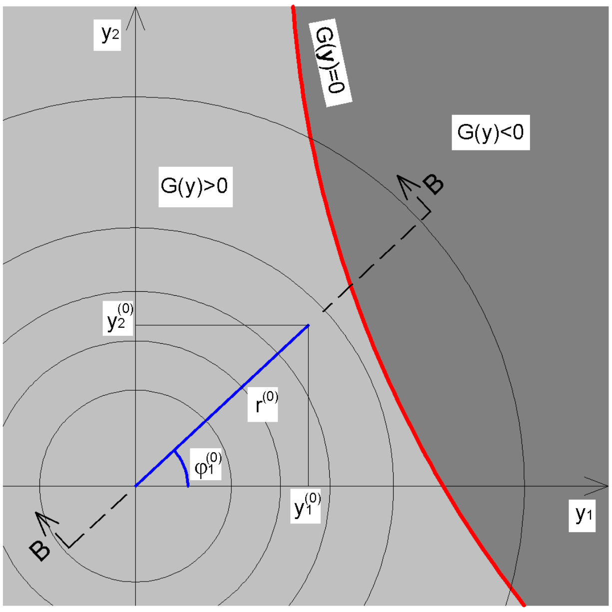


Figure 5.4: Example of application of the standard FORM procedure. Representation of the starting vector $\mathbf{y_0}$ in the standardized Gaussian space.

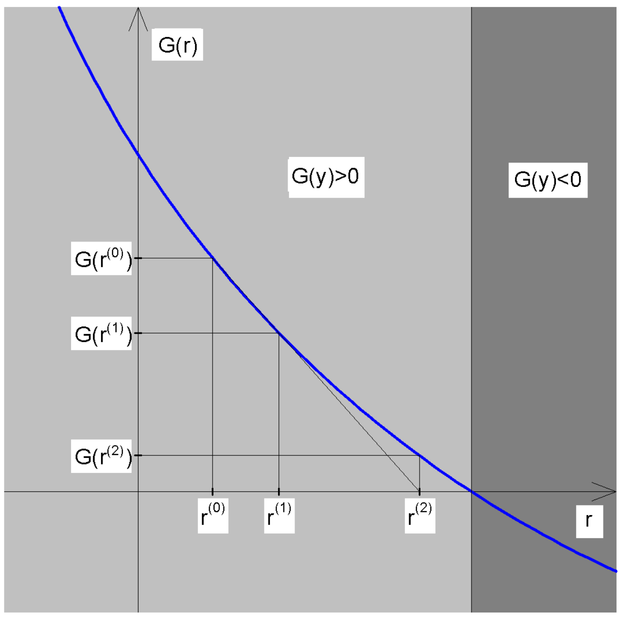


Figure 5.5: Section B-B of figure 5.4. Example of Newton's method application.

Figure 5.4 represents the starting vector in the standard Gaussian space. There is an area where $G(\mathbf{y})<0$ (failure) and one where $G(\mathbf{y})>0$ (success). The first target is to determine where the critical condition $G(\mathbf{y})=0$ is reached by moving along the direction of r and keeping φ_1 fixed.

If figure 5.4 is "sliced" along section B-B, figure 5.5 is obtained.
At first the critical point $G(r)=0$ is approached with Newton's method. When two consecutive values of G have opposite sign (i.e., $G(r^{(i)})\cdot G(r^{(i-1)})<0$) the algorithm switches to bisection method (figure 5.6).
The algorithm goes on until the stop criterion $|r^{(i+1)}-r^{(i)}|<r_{tolerance}$ is satisfied.

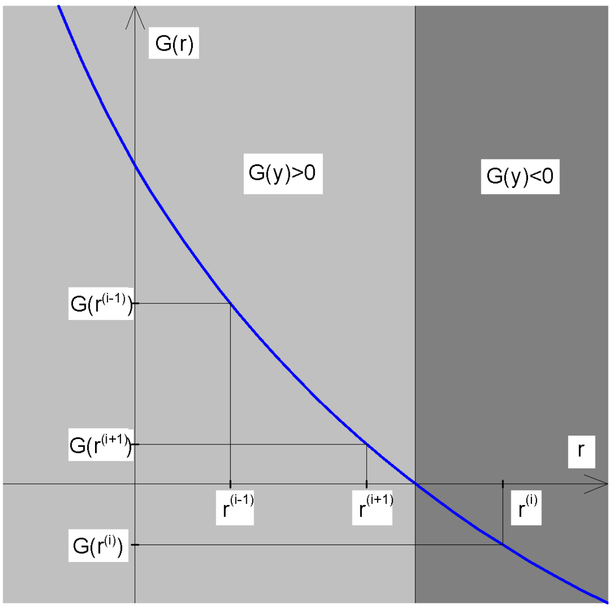


Figure 5.6: Section B-B of figure 5.4. Example of bisection method application.

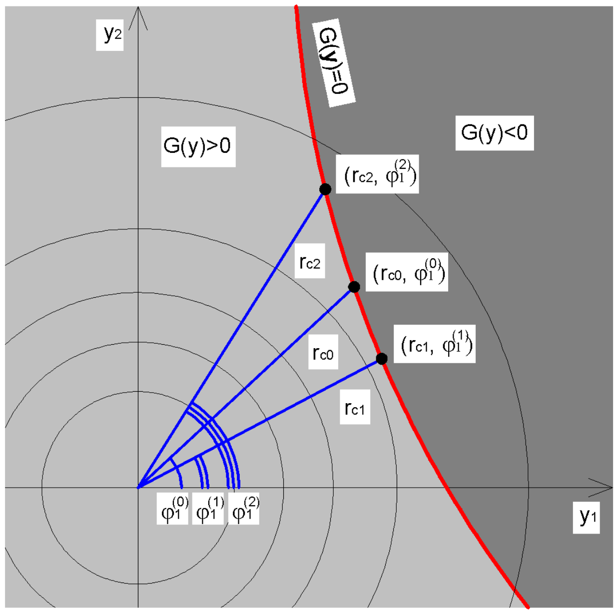


Figure 5.7: Example of application of the standard FORM procedure. Representation of the first three calculated critical points in the standardized Gaussian space.

Once the critical value r_{c0} is obtained, the φ_1 coordinate is slightly changed and another critical value r_{c1} is calculated. Then φ_1 is changed one more time and a third critical value r_{c2} is calculated (figure 5.7).

Then, the function $r(\varphi_1)$ given the condition $G(r,\varphi_1)=0$ is minimized through successive parabolic interpolation, i.e. fitting a parabola at three points of the function, taking the minimum of the fitted parabola as a new point and discarding the "old" point in which the value of the function is higher, repeating until a stop criterion is reached.

With reference to figure 5.8, the parabola that goes through the three critical points $(r_{c0},\varphi_1^{(0)})$, $(r_{c1},\varphi_1^{(1)})$, $(r_{c2},\varphi_1^{(2)})$ is calculated. The value $\varphi_1^{(new)}$ corresponding to the minimum of the parabola is taken, while the point with the higher value of r is discarded.
The critical value of r corresponding to $\varphi_1^{(new)}$ is

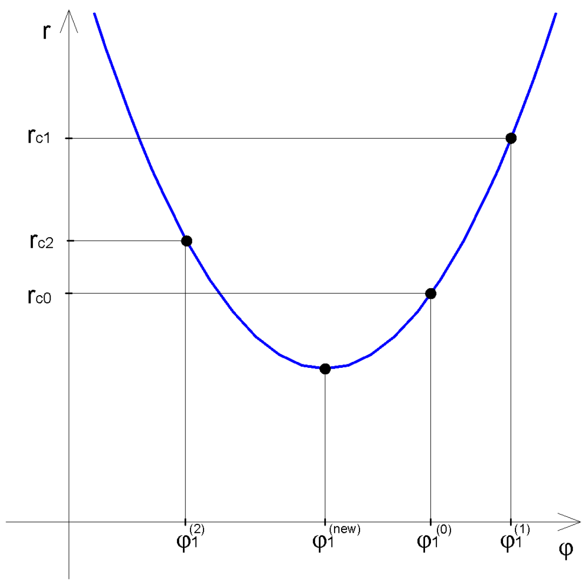


Figure 5.8: Example of parabolic interpolation.

calculated and the parabola minimization is repeated with the new set of points. This step is reiterated until a stop criterion is satisfied.

In the 2-dimensional example, the resulting value of r corresponds to the reliability index β. If the number of random variables m is higher than 2, then the vector space $\boldsymbol{\varphi}$ has dimension m-1 and minimization must be performed along m-1 conjugate directions.

In the conjugate direction method, a basis of the $\boldsymbol{\varphi}$ space is arbitrarily chosen. In the example illustrated in figure 5.9, $\boldsymbol{\varphi} \in R^2$ and the basis is composed of the columns of the identity matrix, i.e. the vectors $\{1,0\}^t$ and $\{0,1\}^t$.
The minimization procedure is performed along all the directions of the basis. If the starting point is labeled P_0 and the point reached after the minimization along the i-th direction is labeled P_i, then after m-1 minimizations the point P_{m-1} is reached (point P2 in figure 5.9). Another minimization is then performed along the direction (P_{m-1}-P_0) (reaching point P3 in figure 5.9).
The basis is then redefined by substituting one of its vectors with the (P_{m-1}-P_0) direction.
The described steps (minimization along m directions and redefinition of the basis) are repeated m-1 times. After a total of m·(m-1) minimizations the elements of the basis are conjugated. If the problem is quadratic, the minimum is reached; otherwise the procedure might need to be performed again.

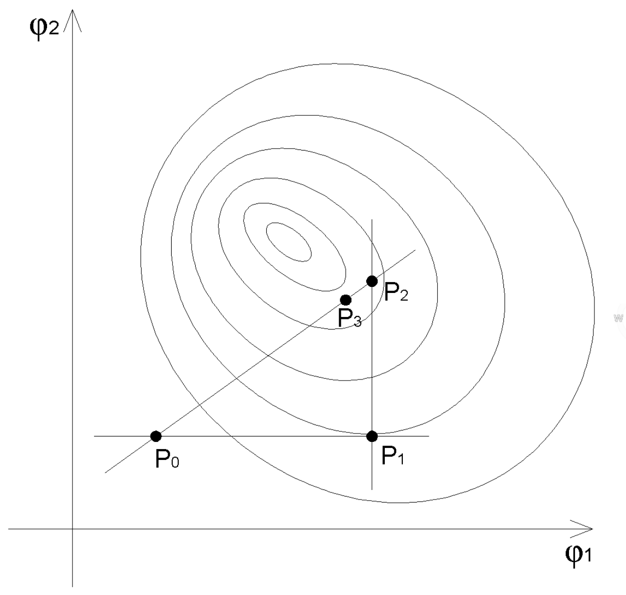


Figure 5.9: Representation of the first steps of the conjugate direction minimization method.

5.3.1.1.4 The simplified FORM procedure

Even though the standard FORM procedure is faster than other methods, it still requires considerable effort. Val et al. ([50], [52]) propose an approximated FORM procedure that reduces drastically the required computational effort. It basically consists in formulating an approximated analytical expression of the resistance R and, consequently, of the performance function g(**x**); then the probability is calculated with gradient methods.
This FORM procedure is more approximated than the standard one, but it is much faster; with static analysis the characterization of R requires only one model run. Furthermore, in the tests reported by the Authors, as well as in those performed in this work the approximation always resulted on the safe side (i.e., overestimating the probability of failure P_f).

For the resistance R a Gaussian distribution is used. A deterministic analysis is performed, in which all random variables controlling the resistance are set to their mean value. The resulting Collapse load is is taken as the mean value of the resistance μ_R.
The coefficient of variation[2] of the resistance V_R is taken as that of the most important variable controlling the resistance. In the examples reported by the Authors, where reinforced concrete structures are analyzed, this corresponds to the yield strength of steel reinforcement when the failure mode of the structure has predominantly plastic behavior and to the compressive strength of

2 The coefficient of variation of a random variable is defined as the ratio between its standard deviation and its mean value: V_i=σ_i/μ_i.

the concrete when the failure mode has predominantly brittle behavior. In the examples implemented in this work, where steel frame structures are studied, the yield strength of the material is used.

The adopted gradient method is the Rackwitz-Fiessler algorithm [41], which is implemented on a worksheet (figure 5.10).

Matodo semplificato di Val per il calcolo del Reliability Index
Xi, R, D=Gauss
L=Gumbel

Parametro	Media	Deviazione standard	Parametro	Media	Deviazione standard	Coeff. Di variazione				
Xi	1	0.075	Xi	1	0.075	0.075	mi_fy	397'000'000	sigma_L	960
xR	1.74664	0.14079	R	22'102	1'782	0.0806	sigma_fy	32'000'000	mi_D	10'255
xD	0.81042	0.04078	D	10'255	516	0.050	V_fy	0.080605	sigma_D	516
xL	0.18958	0.07587	L	2'399	960	0.400	mi_L	2'399	R	22'101.93
									xR	1.746636

Passo	0	1	2	3	4	5	6	7	8	9	10
y1	0	-2.266	-2.065	-1.825	-1.726	-1.690	-1.678	-1.674	-1.672	-1.672	-1.672
y2	0	-2.435	-2.292	-2.033	-1.915	-1.870	-1.854	-1.849	-1.847	-1.846	-1.846
y3	0	0.705	0.800	0.697	0.643	0.622	0.615	0.613	0.612	0.612	0.611
y4	0	1.255	1.994	2.317	2.464	2.523	2.543	2.550	2.553	2.554	2.554
g(y)	0.7595	0.0409	-0.0168	-0.0043	-0.0007	-0.0001	0.0000	0.0000	0.0000	0.0000	0.0000
Delta_g1	0.13100	0.10528	0.10680	0.10953	0.11077	0.11125	0.11142	0.11148	0.11149	0.11150	0.11150
Delta_g2	0.14079	0.11686	0.11899	0.12152	0.12256	0.12294	0.12307	0.12311	0.12313	0.12313	0.12313
Delta_g3	-0.04078	-0.04078	-0.04078	-0.04078	-0.04078	-0.04078	-0.04078	-0.04078	-0.04078	-0.04078	-0.04078
Delta_g4	-0.07254	-0.10166	-0.13559	-0.15633	-0.16534	-0.16865	-0.16978	-0.17015	-0.17027	-0.17031	-0.17033
Delta_g^2	0.04391	0.03674	0.04561	0.05287	0.05629	0.05760	0.05805	0.05820	0.05825	0.05826	0.05827
Moltiplicatore	-17.298	-19.610	-17.086	-15.761	-15.257	-15.081	-15.022	-15.003	-14.997	-14.994	-14.994
Beta	**0.00000**	**3.62463**	**3.75870**	**3.64901**	**3.62385**	**3.61989**	**3.61940**	**3.61934**	**3.61934**	**3.61934**	**3.61934**
x1	1.000	0.830	0.845	0.863	0.871	0.873	0.874	0.874	0.875	0.875	0.875
x2	1.747	1.404	1.424	1.460	1.477	1.483	1.486	1.486	1.487	1.487	1.487
x3	0.810	0.839	0.843	0.839	0.837	0.836	0.835	0.835	0.835	0.835	0.835
x4	0.190	0.268	0.360	0.421	0.449	0.460	0.463	0.464	0.465	0.465	0.465
esponente	0.577	1.898	3.461	4.491	4.963	5.141	5.202	5.222	5.229	5.231	5.232
F(x4)	0.570	0.861	0.969	0.989	0.993	0.994	0.995	0.995	0.995	0.995	0.995
Fi^-1(F(x4))	0.17727	1.08429	1.86758	2.28515	2.45892	2.52196	2.54336	2.55044	2.55276	2.55352	2.55377
f(x4)	5.41405	2.18010	0.51439	0.18748	0.11738	0.09836	0.09255	0.09070	0.09010	0.08990	0.08984
Dev Standard	0.07254	0.10166	0.13559	0.15633	0.16534	0.16865	0.16978	0.17015	0.17027	0.17031	0.17033
Media	0.17673	0.15752	0.10694	0.06383	0.04246	0.03422	0.03136	0.03041	0.03009	0.02999	0.02996
Xi	1.000	0.830	0.845	0.863	0.871	0.873	0.874	0.874	0.875	0.875	0.875
R	22'102	17'763	18'019	18'480	18'690	18'771	18'799	18'808	18'811	18'812	18'813
D	10'255	10'619	10'668	10'615	10'587	10'576	10'572	10'571	10'571	10'571	10'571
L	2'399	3'388	4'558	5'328	5'682	5'815	5'861	5'876	5'881	5'883	5'883

Iteration	Beta
0	0.000000
1	3.624630
2	3.758698
3	3.649014
4	3.623851
5	3.619893
6	3.619400
7	3.619344
8	3.619338
9	3.619337
10	3.619337

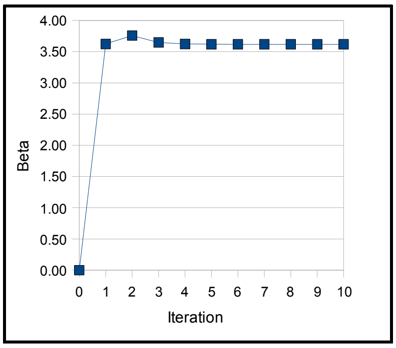


Figure 5.10 : (Left) Worksheet of the Rackwitz-Fiessler algorithm. (Right) Table and diagram of the convergence of β.

The basic principle of gradient methods is to generate a sequence of trial points $\mathbf{y}_1$, $\mathbf{y}_2$, ..., in the standardized Gaussian space according to the rule

$$\mathbf{y}_{k+1}=\mathbf{y}_k+s\mathbf{d}_k \tag{5.13}$$

where $\mathbf{d}_k$ is the search direction vector and s is a parameter defining the step length.
In the Hasofer-Lind Method, at each step the direction vector is defined by

$$\mathbf{d}_k= \frac{[\mathbf{y}_k^t\nabla G(\mathbf{y}_k)-G(\mathbf{y}_k)]\ \nabla G(\mathbf{y}_k)-\mathbf{y}_k}{|\nabla G(\mathbf{y}_k)|^2} \tag{5.14}$$

where $\nabla G(\mathbf{y}_k)$ is the gradient vector of the performance function and s equals unity.
This method was later extended by Rackwitz and Fissler to include the case of non-Gaussian random variables, which are locally approximated to Gaussian distributions in which the values of probability density function and cumulative distribution function is equal to the values of the original distribution.

5.3.1.2 Number of LD levels and LD scenarios to consider

To obtain the actual probability of Collapse P(C=1) using equation (5.4), in theory all the possible Local Damage scenarios need to be considered.
The chosen convention is to quantify the Local Damage level as the number of “failed” structural

elements, so in a structure with n elements:

- there are n+1 Local Damage levels (from 0 to n);
- the number of scenarios for the i-th LD level is $\binom{n}{i}=\frac{n!}{i!(n-i)!}$;
- and the total number of scenarios is $\sum_0^n{}_i \binom{n}{i}=2^n$.

In reality, in oder to have an effective decision tool it is not necessary to consider so many terms, as the next two sections illustrate.

5.3.1.2.1 Local Damage levels

For the sake of simplicity, in the following explanation equation (5.3) is used instead of equation (5.4), but the concepts are equally valid.
If the Structural Vulnerability terms $P(C=1|LD_i)$ are calculated up to the Damage Level p, with p<n, then equation (5.3) can be rewritten as

$$P(C=1)=\sum_0^p{}_i P(C=1|LD_i)P(LD_i)+\sum_{p+1}^n{}_i P(C=1|LD_i)P(LD_i) \qquad (5.15)$$

where the value of the first summation is known and the second is unknown.
Since $0 \leq P(C=1|LD_i) \leq 1\ \forall i$, the second summation is a number comprised between 0 and $\sum_{p+1}^n{}_i P(LD_i)$. As a consequence, the exact value of P(C=1) is surely comprised between

$$P(C=1)_{min}=\sum_0^p{}_i P(C=1|LD_i)P(LD_i)+0$$

and

$$P(C=1)_{max}=\sum_0^p{}_i P(C=1|LD_i)P(LD_i)+\sum_{p+1}^n{}_i P(LD_i) .$$

Thus, for each value of p you get to know an interval in which P(C=1) is situated; moreover, you can estimate P(C=1) and calculate the maximum approximation error of the estimate. Of course, as p increases, the interval and the maximum error decrease.
In particular,

- if the estimate $P(C=1)=[P(C=1)_{max}+ P(C=1)_{min}]/2$ is taken, then
  - the maximum absolute approximation error is $[P(C=1)_{max}- P(C=1)_{min}]/2=\Delta P(C=1)/2$
  - and the maximum relative approximation error is $\Delta P(C=1)/[2 \cdot P(C=1)_{min}]$.
- If instead it is assumed $P(C=1)=P(C=1)_{min}$ or $P(C=1)=P(C=1)_{max}$, then
  - the maximum absolute approximation error is $\Delta P(C=1)$
  - and the maximum relative approximation error is $\Delta P(C=1)/P(C=1)_{min}$.

If it has already been established a target value $P(C=1)_{acc}$ (i.e., a value of P(C=1) that the studied structure must not exceed), then:

- If $P(C=1)_{max}$ is lower than $P(C=1)_{acc}$, then P(C=1) is also lower and the structure is acceptable; there is no need to calculate other terms of the equation.
- Conversely, if $P(C=1)_{min}$ is higher than $P(C=1)_{acc}$, then P(C=1) is also higher and the structure is not acceptable.
- If $P(C=1)_{min}<P(C=1)_{acc}<P(C=1)_{max}$, then further calculations are needed. If you do not want or

cannot make them, you still have the estimate of P(C=1) and of its approximation error.

Furthermore, if the structure is deemed not acceptable, the calculated Structural Vulnerability terms $P(C=1|LD_{ik})$ will be useful to identify the parts of the structure where an intervention is more effective in order to reduce the Structural Risk P(C=1).

LD	$P(LD_i)$	$P(C=1\|LD_i)$	$P(C_i=1)$	$P(C=1)_{min}$	$P(C=1)_{max}$	$P(C=1)_{max}-P(C=1)_{min}$
0	1.00E+000	1.00E-09	1.00E-009	1.00E-09	2.59E-07	2.58E-07
1	1.80E-007	0.19	3.42E-008	3.52E-08	1.13E-07	7.80E-08
2	5.20E-008	0.31	1.61E-008	5.13E-08	7.73E-08	2.60E-08
3	1.60E-008	0.74	1.18E-008	6.32E-08	7.32E-08	1.00E-08
4	1.00E-008	1.00	1.00E-008	7.32E-08	7.32E-08	0.00E+00

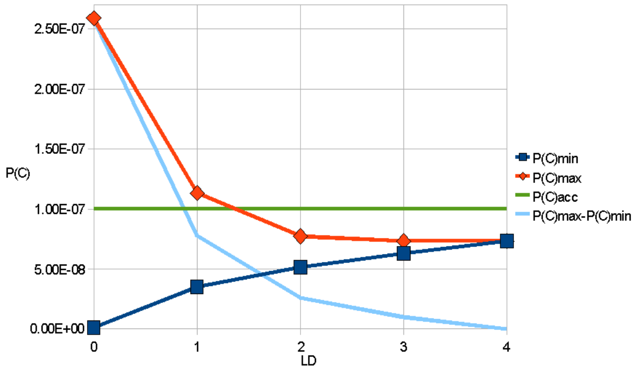


Figure 5.11 : A simple example to illustrate the concepts presented in this section. The parameters reported in the table are fictitious (not calculated), just for the sake of the example. The structure is composed of n=4 elements. It can be seen that, after the calculation of $P(C=1|LD_i)$ up to i=2, it is ascertained that P(C=1) is below the target value.

5.3.1.2.2 Local Damage scenarios

In a structure with n elements there are $\binom{n}{i}=\frac{n!}{i!(n-i)!}$ combinations for the i-th Local Damage level. But it is extremely unlikely that a Hazard will damage structural elements that are distant without damaging the ones located in between. Thus, in the proposed methodologies only the combinations of elements that are located close to each other are considered in the calculation of Risk.

Further reduction of the number of calculations can be obtained by taking advantage of symmetries in the structure.

Figure 5.12 illustrates these concepts.

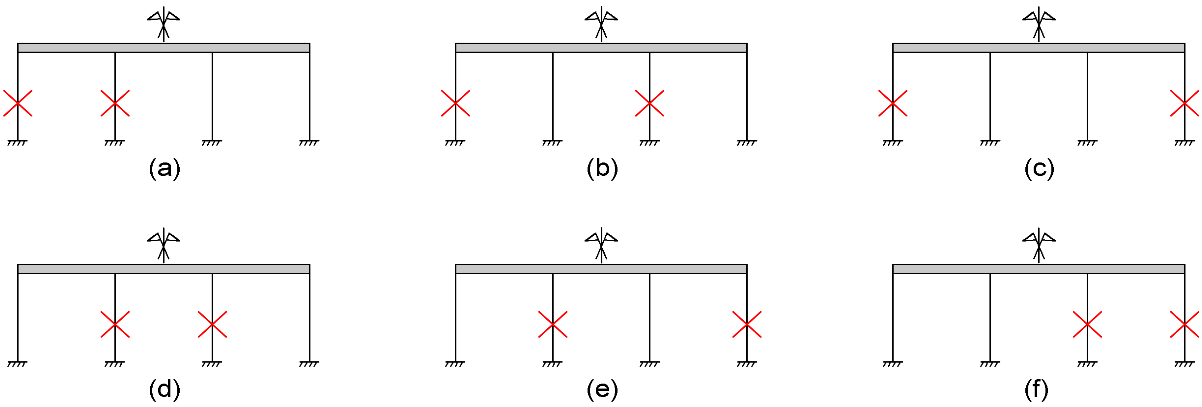


Figure 5.12 : A very simple example to illustrate the concepts of this section. Assuming that in this structure only the four columns can fail, there are six possible Damage scenarios for Local Damage level LD=2. Scenarios (b), (c) and (e) can be excluded because the damaged elements are not close to each other. Scenarios (a) and (f) are symmetrical. Thus, only two scenarios need to be investigated.

5.3.1.3 Evaluation of the $P(LD_{ik})$ terms

Each term $P(LD_{ik})$ is the probability of occurrence of the initial Local Damage scenario k of intensity i. To apply the presented methodologies, at least some of these terms need to be estimated.

In reality, at present time, a good description of $P(LD_{ik})$ is not known yet.
In the examples presented in the chapter 6, a conventional one is used, made from the available statistics augmented with fictitious information. Thus, with reference to section 4.3.1, the presented examples belong to category (b), in which "*the system is probabilistically characterized, the event is conventional*".

As stated in section 4.3.3 it is unknown if the the available data is sufficient to adequately characterize the events that can prompt a Progressive Collapse, but there is a consistent amount of information that has not been collected yet. Only by performing extensive surveys and by creating specific databases it will be possible to ascertain if Progressive Collapse can be included in category (a), where a "real" Risk is calculated.

5.3.1.3.1 Collecting the data

In this section, some basic ideas on how to estimate the $P(LD_{ik})$ terms are presented. These ideas are very simplified, and are just meant to give a hint on how a suitable description of $P(LD_{ik})$ could be obtained. Many aspects must be further studied, and experiments could be needed.

Every time a traumatic event happens, information about it should be collected. This can be done by collecting data about the characteristics of the Hazard and/or about the direct Local Damage that it provokes.

If the characteristics of the Damage are surveyed, the information to look for should be:

- the type of Hazard that provoked it;
- the extension of the Damage directly provoked by the Hazard;
- construction technology of the structure (steel, concrete, masonry...);
- the type of use of the structure (because some structures might be more exposed than others to specific Hazards);
- the area of the structure where the Damage occurred (e.g., lower/higher floors, external/internal bays, areas where gas systems are located, and so on).

This way, if the number of surveyed events is sufficiently high, an estimate of $P(LD_i)$ for several Hazards could be obtained, and for different types of structures. Unlike the currently available statistics, these would consider not only the occurrence of Damage, but also its extension. In other words, this methodology should provide a set of values $\{P(LD=1), P(LD=2), ..., \}$ for each surveyed type of Hazard. Furthermore, the survey should highlight the areas of the structures where the Damage is more likely to happen, because with some Hazards some elements are more exposed then others. With other Hazards (like for example material corrosion, material defects, misuse of the structure) it can be assumed that the exposure is the same for all areas of the structure.

Alternatively, the characteristics of the Hazard can be collected. In this case the information to look for should be:

- the type of Hazard;
- its intensity;
- construction technology of the structure;
- the type of use of the structure;
- the area of the structure where the Hazard occurred.

This way the probability of occurrence of each Hazard could be estimated. Again, the estimates would consider not only the occurrence of the Hazard, but also its intensity, which can be expressed with one of the characteristic parameters of the Hazard. For example, for explosions the peak overpressure can be used; for vehicle impact the kinetic energy of the vehicle, and so on.

To link the characterization of the Hazard P(H) to the characterization of the Local Damage P(LD), in theory a probabilistic study is required, using the relation

$$P(LD)=P(LD \cap H)=P(LD|H)P(H)$$

This would require calculating the P(LD|H) terms, i.e. the probability of occurrence of a given Local Damage level for different intensities of the Hazard. To avoid the effort of probabilistic analyses, a deterministic approach could be adopted, by assuming that a structural element of given characteristics (geometry, materials) will fail if the Hazard intensity acting on it is equal or higher than a critical value HI_{crit}, and will not be damaged otherwise.

By applying the presented ideas, the contribution of several Hazards to the total value of $P(LD_{ik})$ can be figured out, at least for some values of i and k. Furthermore, since most Progressive Collapses have happened because of unforeseen Hazards or because of Hazards of bigger intensity than expected, it should be wise to take in account these unexpected events by adding a default contribution to $P(LD_{ik})$.
Once all these contributions to $P(LD_{ik})$ are estimated, they must be combined together. In general, given two independent events A and B, the probability of occurrence of at least one event is

$$P(A \cup B)=P(A)+P(B)-P(A) \cdot P(B)$$

Since in the considered context the product $P(A) \cdot P(B)$ is of a smaller order of magnitude compared to the other terms, it can be neglected with minimal approximation. Thus $P(LD_{ik})$ can be assumed as the sum of the contribution of all the Hazards. For example, it could be:

$$P(LD_{ik})=P(LD_{ik})_{\text{gas explosion}}+P(LD_{ik})_{\text{vehicle impact}}+P(LD_{ik})_{\text{deterioration}}+P(LD_{ik})_{\text{misuse}}+P(LD_{ik})_{\text{unexpected event}}$$

5.3.2 Implementing the second methodology

The second methodology differs from the first one in that it considers the final extension of the Damage. In order to calculate the Structural Risk $P(FD=f) \cdot f$ using equation (5.5) three main elements are needed:

- the Structural Vulnerability terms $P(FD=f|LD_{ik})$ need to be calculated;
- the $P(LD_{ik})$ terms must be evaluated;
- it must be assessed how many Local Damage scenarios need to be considered.

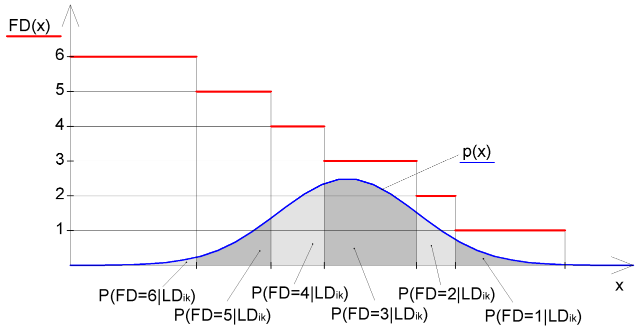
The second and third point have already been discussed in sections 5.3.1.2 and 5.3.1.3, respectively. The following section deals with the first point.

5.3.2.1 Calculation of the Structural Vulnerability

In the second methodology, the defined Structural Vulnerability $P(FD=f|LD_{ik})$ is the probability that a Collapse occurs, whose final extension corresponds to the failure of f structural elements, after the structure has suffered a Local Damage scenario LD_{ik}.

Like with the first methodology, this probability can be calculated with reliability methods.
The conceptual difference between the two methodologies is that, while in the first one the basic problem to solve consisted in identifying the sets of random variables for which a performance function $g(\mathbf{x})$ is negative or positive, in the second one the goal is to identify the sets of random

variables for which the Final Damage level FD assumes a given value f.
This section explains how this can be done.



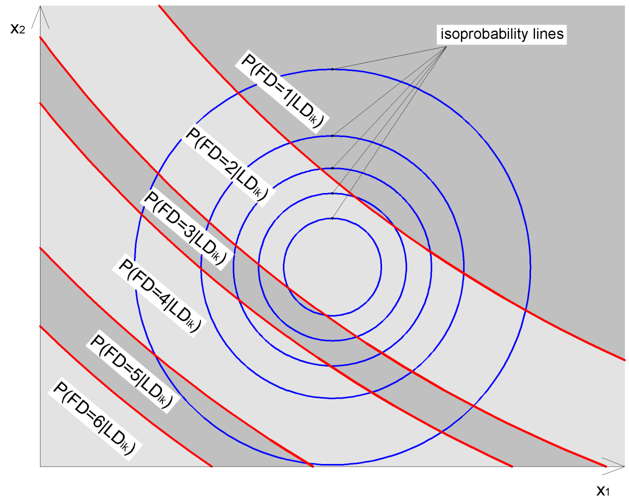


Figure 5.13 : Two representations of the probability space of P(FD=f|LD$_{ik}$). (Left) Case of one single random variable. (Right) Case of two random variables.

For simplicity's sake, it is assumed at first that the problem only depends on one random variable x, whose probability density function p(x) is known.
Given an initial Local Damage scenario LD$_{ik}$, if the Final Damage level FD for different values of x is calculated, then a diagram like the one represented in figure 5.13 (left) can be drawn. More specifically, in figure 5.13 (left):

- the horizontal axis represents the random variable x;
- the red lines represent the value of the Final Damage level FD as a function of x, measured on the vertical axis;
- the blue line represents the probability density function p(x);
- the hatched areas are the Structural Vulnerability terms P(FD=f|LD$_{ik}$).

Figure 5.13 (right) represents the same concept in the case of two random variables. Each value of P(FD=f|LD$_{ik}$) is the part of volume of the joint probability density function above one hatched area. In the general case of m random variables, P(FD=f|LD$_{ik}$) is an m+1 dimensional volume.

Calculation of the P(FD=f|LD$_{ik}$) terms requires reliability methods.
As stated in section 5.3.1.1, reliability methods estimate the probability that a system fulfills (or, equivalently, that it does not fulfill) its required performance during a specified period of time under stated conditions.
In the present case the required performance can be defined as the Final Damage level being higher than a given value. This way, calculating a probability P(FD>f|LD$_{ik}$) is conceptually identical than calculating P(C=1|LD$_{ik}$) in the first methodology. Then, the Structural Vulnerability terms will simply be given by the difference of two of such terms, like for example

$$P(FD=3|LD_{ik})=P(FD>2|LD_{ik})-P(FD>3|LD_{ik}).$$

If the trend of FD (as a function of the random variables) is regular enough, then the First Order Reliability Method could be used, but since the problem is highly non linear, it might be not suitable. Thus other methods should be used.
In any case, applying the second methodology would require much more effort than the first one, because it requires modeling the evolution of Collapse, because multiple FD levels are considered and because it likely involves a higher number of random parameters. For these reasons, in the next

chapter only the first methodology is implemented and tested.

5.4 Summary

In this chapter, two methodologies to quantify Progressive Collapse propensity of frame structures are presented.
The motivations to the development of these methodologies, as well as the targets they aim to, are presented in section 5.1.
The basic ideas of the methodologies is presented in section 5.2, and how they can be implemented is explained in section 5.3.

The following chapter presents several examples in which the first proposed methodology is applied.

Chapter 6 - Applying the first methodology

In this chapter, some test applications of the first proposed methodology are presented.
Section 6.1 describes the used structural models.
Section 6.2 presents examples in which the Structural Vulnerability is calculated on three different structures.
Section 6.3 presents two examples of Structural Risk calculation.
Section 6.4 summarizes the chapter.

6.1 Description of the used structural models

The presented examples are implemented using Finite Element models derived from one developed by the National Institute of Standards and Technology (NIST) for an ongoing research program ([20], [21], [42]) aimed at understanding the behavior of structures during Progressive Collapse (figure 6.1).
The models can be used to perform both dynamic and static analyses, with geometrical and material nonlinearities; furthermore, they are able to spot the moment in which a section becomes detached, which corresponds to Collapse condition in the first proposed methodology.
In most studies carried on so far, element behavior derived from seismic engineering studies has been used to model Progressive Collapses. The NIST tests highlight that, in some cases, the actual behavior can be very different. Thus, for certain aspects the used models are advanced. On the other side, they still need improvement under several aspects, which will be highlighted in the text.

6.1.1 The structural details

NIST had an external company design five steel frame buildings according to regulations and common practice of the USA, specifically for this research program [16].

Physical testing of sub-assemblies of the structural components used in these designs is carried out, recreating the failure modes of Progressive Collapse.

The tests are then reproduced with “High Fidelity Connection” models, i.e. with very detailed Finite Element models, using 3D and 2D elements (figure 6.2).

Then simplified models, called “Reduced Coarse Shell Connection” models, are implemented using 2D elements (shown in the bottom right corner of figure 6.3, left).

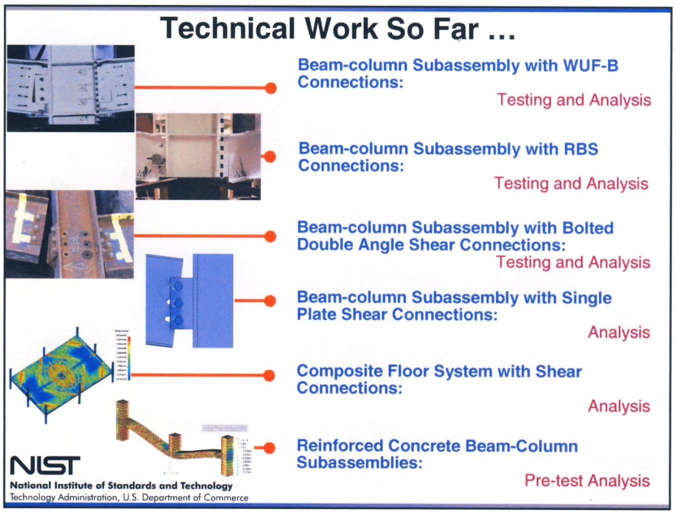


Figure 6.1 : Summary of testing and analysis of structural sub-assemblies carried out by NIST, as of march 2008. (Source: [43]).

Another type of simplified models, called “Reduced Component Connection” is also implemented, in which the connections are modeled as assemblies of very stiff beam elements and elasto-plastic spring elements, while the frame members are modeled with “normal” beam elements (shown in the top left corner of figure 6.3, left).

All the models are implemented with the Finite Element software LS Dyna; its explicit solver has the advantage of having very few convergence problems.
In the end, the models achieve similar results, as shown in figure 6.3 (right).

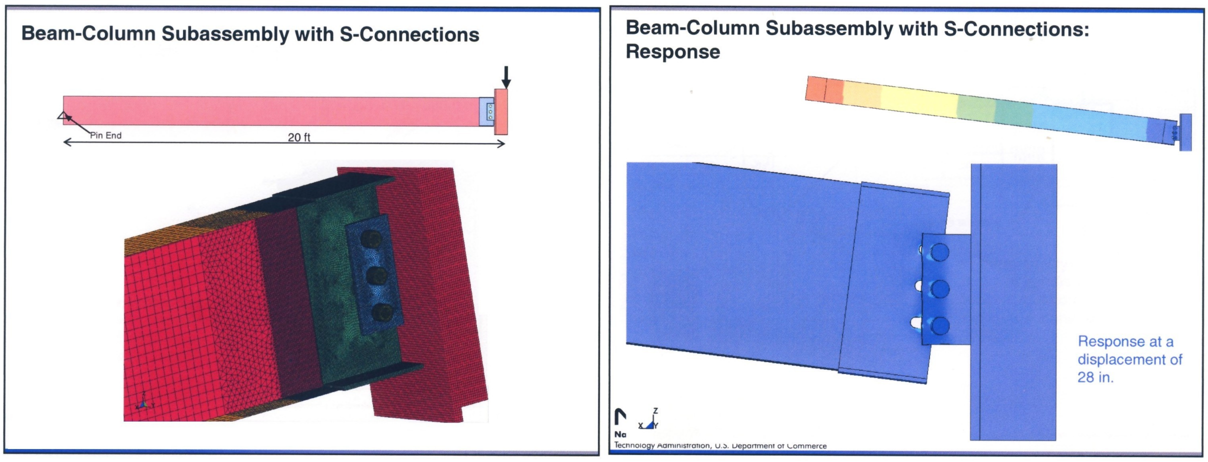


Figure 6.2 : High Fidelity Connection model of a structural sub-assembly. (Elaborated from [43]).

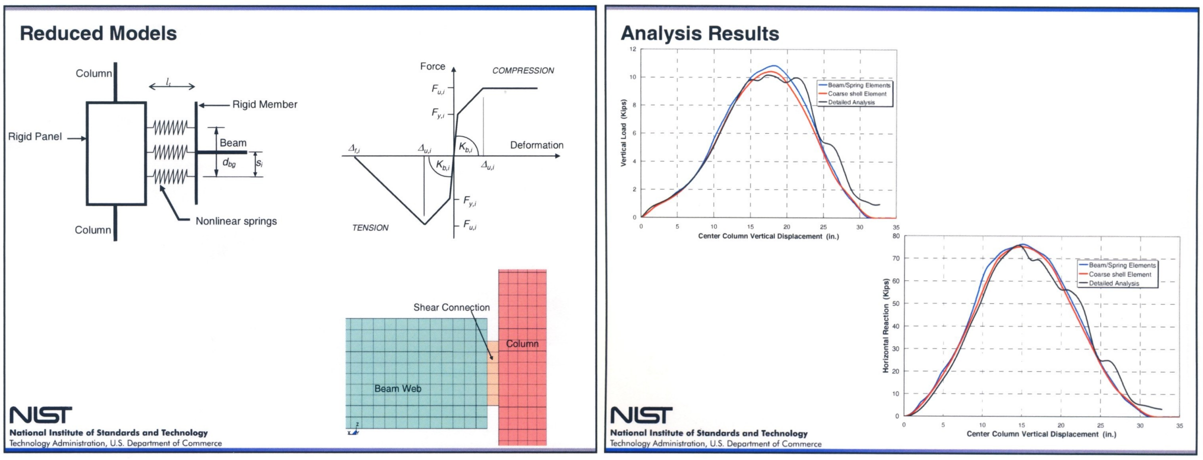


Figure 6.3 : (Left) Reduced Component Connection model (top left corner), Reduced Coarse Shell Connection model (bottom right corner) and simplified force/deformation diagram of a gravity connection. (Right) Comparison of results of the three types of models. (Source: [43]).

The Writer was asked to implement simplified models of two of the steel building designs, to be used in a multi hazard study (wind, earthquake and Progressive Collapse), using the software ANSYS.
In these models the frame members are modeled with normal beam elements, while the connections are modeled with "MultiPoint Constraint" (MPC) elements (i.e. special 0-dimensional elements), which replicate the behavior of the Reduced Component Connection ("stiff beams/spring") assemblies.
With this further simplification, the buildings are modeled in their entirety (previous models only included parts of them). To ensure compliance with the experimental results, models of the structural components with the "stiff beams/spring" connections were replicated with ANSYS. Then, the same components were modeled using MultiPoint Constraint elements.
Studying the connections and developing simplified MultiPoint Constraint elements with an appropriate behavior required considerable time, mainly because of convergence problems (unlike LS Dyna, ANSYS uses an implicit solver).

In common practice in the USA, resistance to lateral loads (such as wind and earthquake) of steel frame buildings is concentrated in a few frames, called moment frames; the other ones are called gravity frames.

During design, resistance to horizontal loads of gravity frames is usually neglected, and their beams are considered hinged to the columns (even though in reality some flexural resistance is always present).
During a Collapse, plasticity develops mainly at the extremes of the beams and in the panel zones (i.e., the part of the column next to the beam joint; see figure 6.4). Thus, the plastic behavior of moment resisting connections, gravity connections and panel zones must be assessed and adequately modeled.

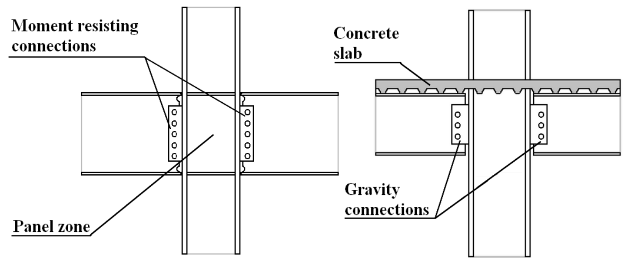


Figure 6.4 : Typical moment resisting connection (left) and gravity connection (right), the latter including a concrete floor slab.

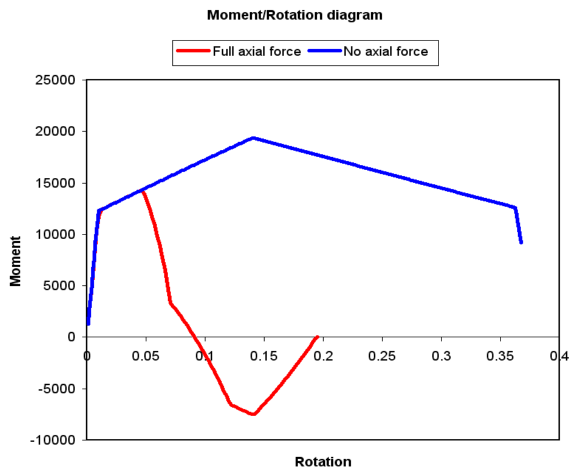


Figure 6.5 : Moment/rotation diagram of a gravity connection with and without axial force.

According to the NIST tests, the actual behavior of structural components under Progressive Collapse can be very different compared to the behavior under earthquake. In particular, beam axial force plays a major role in connections.
As an example, figure 6.5 illustrates the relationship between bending moment and rotation in a gravity connection. The blue line represents the relationship when the influence of the axial force is neglected, the red line represents one case in which it is considered.

More in general, figure 6.6 represents 3D diagrams of bending moment and axial force as functions of rotation (theta) and axial strain (delta) in a gravity connection.

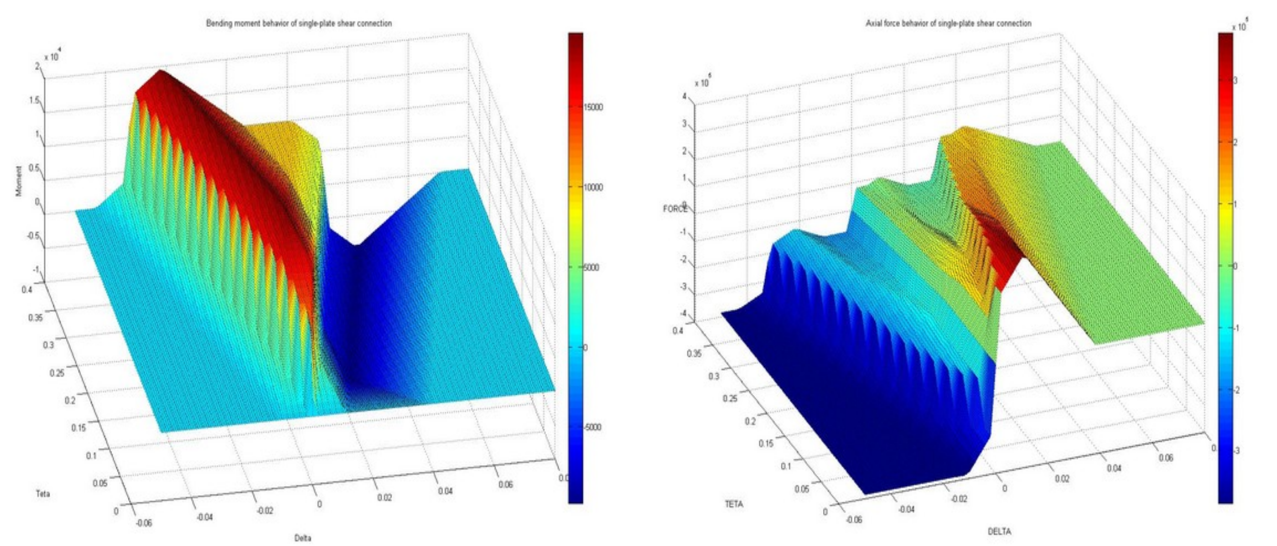


Figure 6.6 : 3-dimensional diagrams of bending moment (left) and axial force (right) of a gravity connection, as a function of its axial strain (delta) and rotation (theta).

Figure 6.7 is a 2-dimensional, “top view” representation of the diagrams of figure 6.6. The lines correspond to the delta-theta (axial strain-rotation) path followed by the connection in different Finite Element analyses. Each line corresponds to a different length of the beam represented on top of figure 6.2.
Diagrams such as those of figure 6.5 correspond to the section of the 3D diagrams with the vertical surface defined by a path line. The path corresponding to the blue line in figure 6.5 is actually not represented in figure 6.7; it corresponds to a straight line, parallel to the theta axis and through the origin.

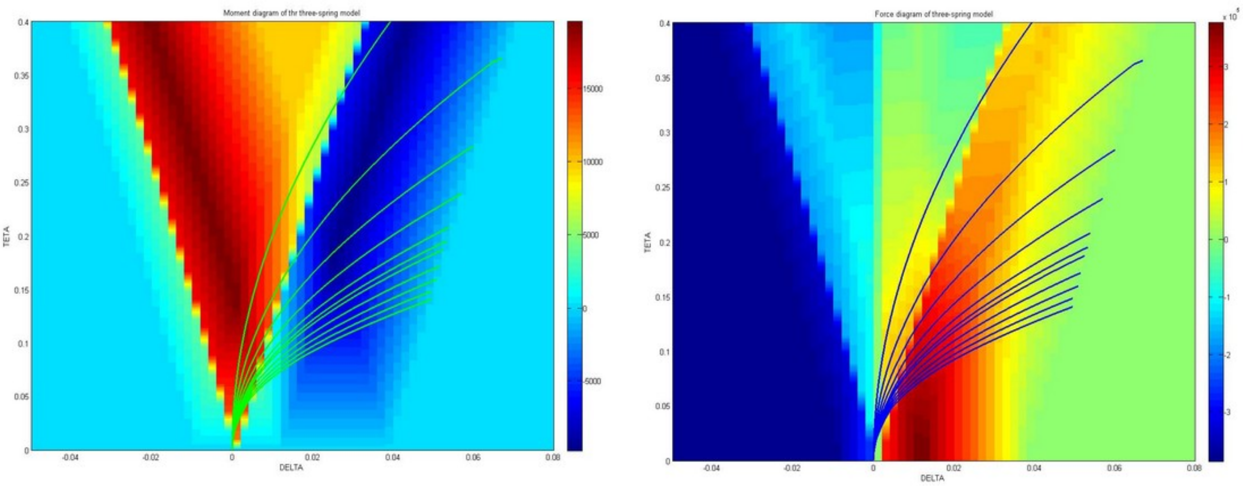


Figure 6.7: 2-dimensional representations of the diagrams shown in figure 6.6 (left: bending moment; right: axial force), with added lines representing the delta-theta paths followed by the connection in different Finite Element analyses.

6.1.2 The designs

Both modeled steel building designs are 10 story 5x5 bay office buildings, with a plan dimension of 45.72 x 30.48m (150' x 100') and a height of 43.053m (141'-3"). The floor systems consist of steel beams and a metal deck with a lightweight concrete topping.

Figure 6.8: Plan layout of the used model. The moment resisting frames are highlighted in orange. (Source: [16]).

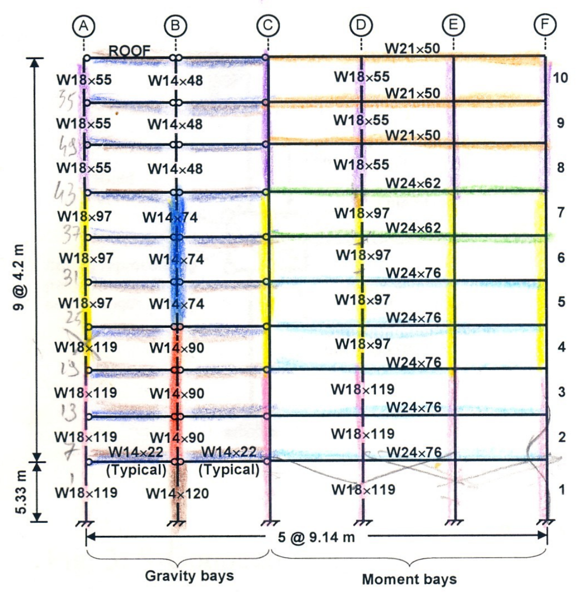


Figure 6.9: Elevation of the used model. Each color corresponds to a frame section type. (Source: [21]).

Figures 6.8 and 6.9 illustrate the plan layout and the elevation of the one used as a basis for this work. It is designed according to the design Standard ASCE 7-02 [5], to be located in Atlanta, Georgia.

All the employed beam section types follow the nomenclature of the American Institute of Steel Construction (AISC) [3].

The lateral force resisting system is composed by Intermediate Moment Frames (IMF), as defined by the AISC Seismic Provisions (2002) [2]. These frames "*are expected to withstand limited inelastic deformations in their members and connections when subjected to the forces resulting from the motions of the Design Earthquake*" [2].

Beam-column connections in gravity frames are single-plate shear tab type, as specified in in AISC LRFD [1]. This type of connection is represented in figure 6.2.

Beam-column connections in moment frames are WUF-B type (Welded Unreinforced Flange-Bolted Web), as specified in FEMA 350 [14]. This type of connection is represented in figure 6.10.

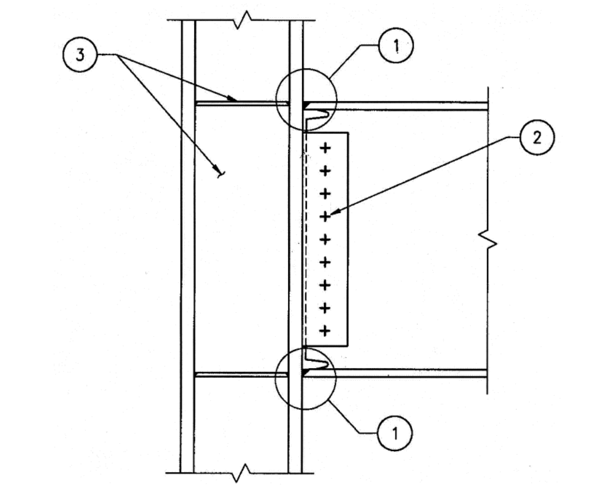


Figure 6.10: Welded Unreinforced Flange – Bolted Web (WUF-B) connection. (1) Both the top and bottom flange of the beam are welded to the column flange. (2) The shear tab is welded to the column flange and bolted to the beam web. (3) Panel zone and continuity plate. (Source: [14]).

The employed materials are A992 structural steel (yielding stress f_y=345 MPa) for beams and columns, and A36 steel (f_y=248 MPa) for shear tabs and continuity plates.

In the implementer models, each beam is divided into 10 segments and each column in 5 segments, to capture the effects of geometrical nonlinearities. This segmentation was adopted because it is common practice at NIST. Further testing is needed to assess if a denser segmentation is needed (for more accuracy) or if coarser would be acceptable (thus making the models smaller and faster).

Beams are modeled with ANSYS' one-dimensional BEAM188 element, with a bilinear isotropic (elastic-perfectly plastic) material behavior (i.e., beam sections can have some or all of their fibers in plasticity).

Panel zones and beam-column connections in gravity frames are modeled with ANSYS' zero-dimensional MPC184 (MultiPoint Constraint) element, to replicate the behavior obtained from the Reduced Component Connection ("stiff beams/spring") models. MPC184 are a general class of elements that apply kinematic

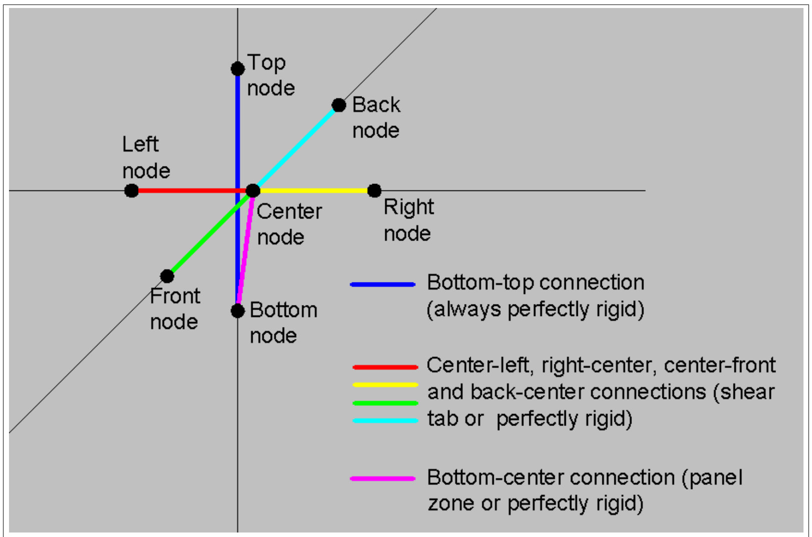


Figure 6.11 : Schematic representation of the arrangement of the MPC184 elements in a generic node of the model. In reality, the elements have no dimension and the seven points are in the very same position.

constraints between nodes. In all, each beam/column node required up to six MPC184 elements, as illustrated in figure 6.11.

Since experimental data about the WUF-B connection were not available, it was assumed that in moment resisting frames the beam-column interface cannot fail (i.e., become detached) and the beam-column connection is modeled as a perfectly rigid joint. However, plastic hinges can form in the BEAM188 elements because of the adopted elastic-perfectly plastic material behavior. The element can reach the ultimate limit state (Collapse condition) when all its fibers have reached yielding point and equilibrium cannot be satisfied anymore. Phenomena like flange instability or fracture are not considered, so the modeled ultimate limit state is conventional.

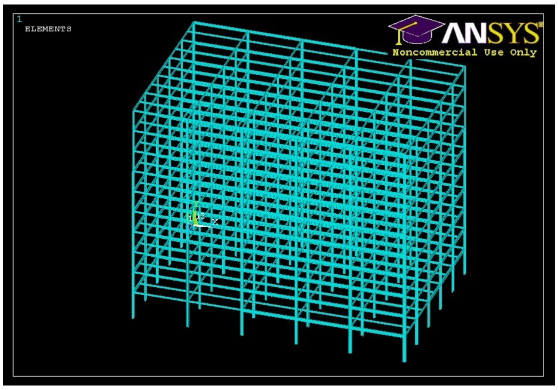


Figure 6.12 : One of the complete 3D models.

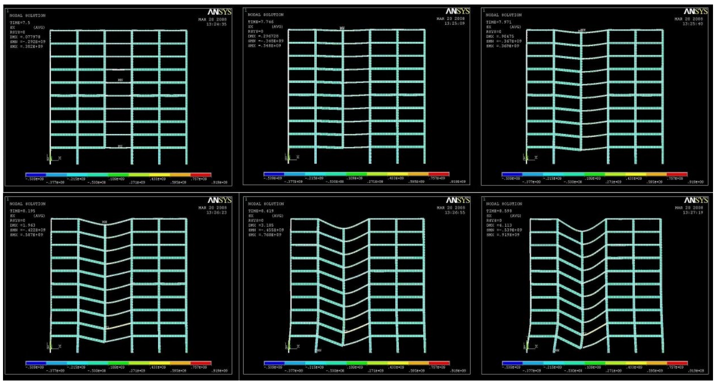


Figure 6.13 : A Collapse sequence of a 2D model.

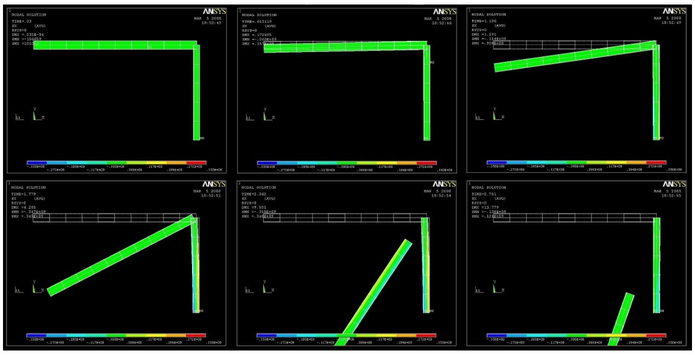


Figure 6.14 : Collapse sequence of a simple structure, as computed by ANSYS.

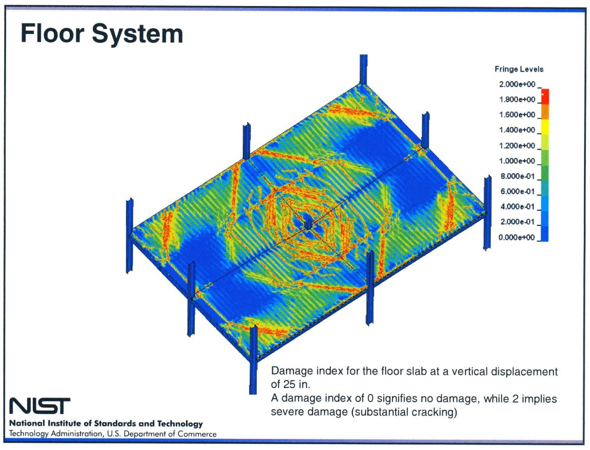


Figure 6.15 : Model of a floor system. (Source: [43]).

The completed models can perform both dynamic and static analyses, with geometrical and material nonlinearities. The models are able to spot the moment in which a section becomes detached. The analysis can go on after the detachment, as shown in figure 6.14, but in this phase the model does not follow the correct laws of physics.

The models still need improvement. In particular, they do not include floor slabs and partitions, which can play an important role in Collapse behavior (as shown in the example of figure 2.2). NIST developed some very detailed models that include floor slabs, using 2- and 3-dimensional elements (figure 6.15), but the considerable number of elements makes them impractical to use. Further research is needed to find a way to realistically replicate the influence of slabs and wall panels in simplified models.

6.2 Structural Vulnerability calculation

In this section the Structural Vulnerability term $P(C|LD_{ik})$ is calculated for several different scenarios on three types of models (a simple portal, a three-story structure and a nine-story structure). For comparison, both FORM procedures (described in section 5.3) are applied to each analyzed scenario.

It must be pointed out that the main target of the performed analyses is not the testing of the structures, but the testing of the algorithms used for the analyses, to study the the feasibility of the proposed approaches, to find out their problems, and to debug the implemented algorithms. Attention is given to the required computation times, because in order to calculate a Structural Risk the Structural Vulnerability must be computed for a large number of Local Damage scenarios.

All the presented models and probabilistic analyses are directly implemented in ANSYS with its parametric language APDL; no other software is used. All input consists in text files, while results are reviewed by exporting them into a spreadsheet program.
Static nonlinear analysis is used to calculate the resistance of the structure. The maximum load is applied in 50 evenly spaced substeps, and the resistance R is taken as the load at the substep in which Collapse condition is occurs. If the maximum load does not provoke Collapse, then the maximum load is doubled and the analysis repeated. If Collapse is reached in less than 25 substeps, the maximum load is halved.
All the computations were carried out on a computer with a 2.40GHz Intel P8600 CPU.

6.2.1 Simple portal

The first tested structure is a simple portal. It was decided to start by studying a very simple structure to test and debug the implemented algorithm itself, as well as to devise suitable worksheets to visualize the results (the output of the probabilistic calculations provided by ANSYS consists merely in columns of numbers).

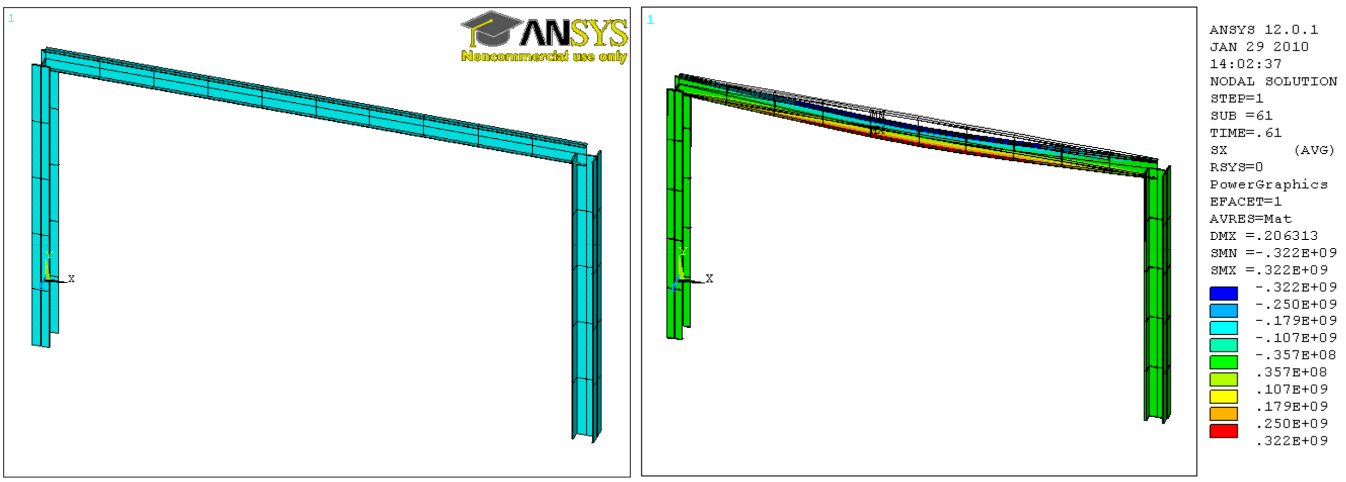


Figure 6.16 : The simple portal structure.(Left) Undeformed shape. (Right) Deformed shape and x-direction stresses at the ultimate limit state (i.e., prior to the detachment of one beam/column connection).

The portal consists of two 5.334m (17'6") long columns connected to a 9.144m (30') long beam. One column is W18X1191 with minimum stiffness orientation, the other column is W14X120 and the beam is W14X22 (as previously stated, all the employed beam section types follow the nomenclature of the American Institute of Steel Construction (AISC) [3]).
Each column is modeled with 5 BEAM188 type elements, while the beam is modeled with 10 elements. The constitutive behavior of steel is linear elastic-perfectly plastic. The beam-column

connections are shear-tab type, each one modeled with one MPC184 element with the behavior obtained from NIST's research.
In all, the model consists of 20 beam type elements and 2 MPC type elements.

Four random variables are considered:

Variable	Type of distribution	Mean value	Standard deviation
Yielding stress of steel fy (N/m^2)	Gauss	3.97E+008	3.20E+007
Live load L (N/m)	Gumbel	2399	960
Dead load D (N/m)	Gauss	10255	516
Model uncertainty ξ	Gauss	1.00	0.075

Table 6.1

The characteristic values of the loads and of the steel are taken from the original design. Their types of distribution, the relationships between characteristic values, mean values, and standard deviations, as well as the characterization of ξ, are taken from Val et al. [50].

To apply the First Order Reliability Method, the performance function

$$g=\xi R-S \tag{6.1}$$

is defined, where

- S=L+D is the total applied vertical load, which is uniformly distributed on the beam;
- R is the Resistance of the structure, i.e. the vertical load that prompts Collapse.

In this particular case, Collapse condition corresponds to the detachment of one beam-column connection (shear-tab connections are always much less resistant than the structural elements they connect). No initial Damage is introduced.

6.2.1.1 The standard FORM procedure

The probability of failure P_f of the portal is calculated with the standard FORM procedure, as described in section 5.3.1.1.1. As stated, the solution of the problem corresponds to finding the reliability index β, which corresponds to the minimum value of the polar coordinate r (radius) given the critical condition $G(r,\boldsymbol{\varphi})=0$ (equation 5.12).

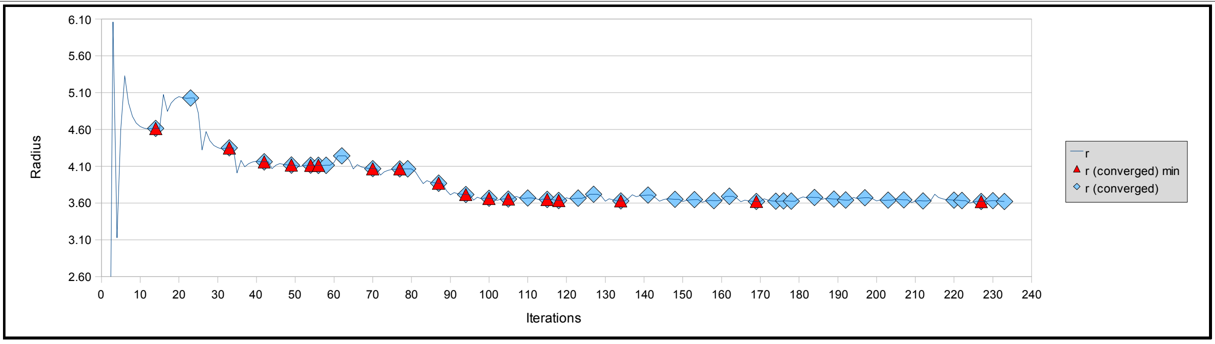
Figure 6.17 illustrates one analysis run.
In the top diagram:

- the horizontal axis represents the number of iterations;
- the vertical axis represents the value of the radius r;
- the blue line represents all the computed values of r;
- the blue squares represent converged values, in which $G(r,\boldsymbol{\varphi})=0$;
- the orange triangles represent the minimum value of r, as the computation progresses.

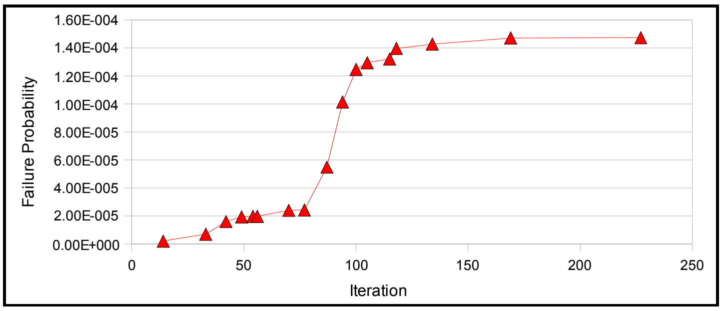
The middle diagram represents the value of the estimated failure probability, as the computation progresses.

The bottom diagram illustrates the first minimization of r along the direction φ_1, including the first four calculated parabolas (as explained in section 5.3.1.1.3).

The table summarizes the parameters of the analysis run.



Iteration	Phi1	Phi2	Phi3	r (converged)	Failure Probability
14	0.000000	0.000000	1.570796	4.613126	1.98E-006
23	0.157080	0.000000	1.570796	5.027586	2.48E-007
33	-0.157080	0.000000	1.570796	**4.348063**	**6.87E-006**
42	-0.357233	0.000000	1.570796	**4.159601**	**1.59E-005**
49	-0.482646	0.000000	1.570796	**4.115431**	**1.93E-005**
54	-0.517214	0.000000	1.570796	**4.112670**	**1.96E-005**
56	-0.523387	0.000000	1.570796	**4.111290**	**1.97E-005**
58	-0.558164	0.000000	1.570796	4.111980	1.96E-005
62	-0.523387	-0.157080	1.570796	4.240522	1.12E-005
70	-0.523387	0.157080	1.570796	**4.065865**	**2.39E-005**
77	-0.523387	0.163678	1.570796	**4.063136**	**2.42E-005**
79	-0.523387	0.320757	1.570796	4.064501	2.41E-005
87	-0.523387	0.163678	1.727876	**3.868284**	**5.48E-005**
94	-0.523387	0.163678	1.884956	**3.715607**	**1.01E-004**
100	-0.523387	0.163678	2.375049	**3.663124**	**1.25E-004**
105	-0.523387	0.163678	2.217969	**3.653284**	**1.29E-004**
110	-0.605601	0.189388	2.319627	3.666984	1.23E-004
115	-0.441174	0.137967	2.116312	**3.648135**	**1.32E-004**
118	-0.432775	0.135341	2.105927	**3.633752**	**1.40E-004**
123	-0.432775	0.292420	2.105927	3.665548	1.23E-004
127	-0.432775	-0.021739	2.105927	3.717929	1.00E-004
134	-0.432775	0.170815	2.105927	**3.628260**	**1.43E-004**
141	-0.432775	0.170815	1.948847	3.707605	1.05E-004
148	-0.432775	0.170815	2.263006	3.649223	1.32E-004
153	-0.432775	0.170815	2.151638	3.645575	1.33E-004
158	-0.514988	0.196525	2.207584	3.632039	1.41E-004
162	-0.350561	0.145104	2.004269	3.690934	1.12E-004
169	-0.469206	0.182208	2.150974	**3.620349**	**1.47E-004**
174	-0.474929	0.189570	2.158050	3.624875	1.45E-004
176	-0.463484	0.174846	2.143898	3.627137	1.43E-004
178	-0.469778	0.182944	2.151682	3.626006	1.44E-004
184	-0.469206	0.182208	2.308054	3.675777	1.19E-004
189	-0.469206	0.182208	1.993895	3.654002	1.29E-004
192	-0.469206	0.182208	2.131776	3.640455	1.36E-004
197	-0.386993	0.156498	2.049316	3.672309	1.20E-004
203	-0.551420	0.207918	2.252632	3.637945	1.37E-004
207	-0.489515	0.188559	2.176086	3.642240	1.35E-004
212	-0.474929	0.189570	2.158050	3.628717	1.42E-004
220	-0.463484	0.174846	2.143898	3.638065	1.37E-004
222	-0.470232	0.183527	2.152242	3.633391	1.40E-004
227	-0.474929	0.189570	2.158050	**3.619901**	**1.47E-004**
230	-0.480651	0.196932	2.165126	3.631145	1.41E-004
233	-0.472287	0.186171	2.154783	3.621229	1.47E-004



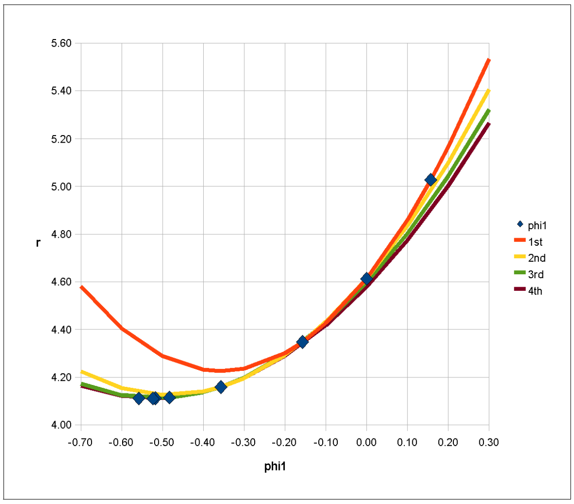


Figure 6.17 : Data of a standard FORM analysis run. (Top) Diagram of the computed values of the radius r. (Middle) Diagram of the corresponding failure probabilities. (Bottom) First parabolic minimizations along the direction φ_1. The table summarizes the parameters of the analysis run.

The computation was tested on several variations of the model. By changing the tentative initial values of the random variables (but no other parameters), the results vary slightly, as shown in table 6.2.

The variation of the results is attributable to the stop criterion in the calculation of the radius r: once the condition $|r^{(i+1)}-r^{(i)}|<r_{tolerance}$ is satisfied, the last calculated value $r^{(i+1)}$ is taken as the converged value, while the actual critical value is comprised between $r^{(i+1)}$ and $r^{(i)}$ but unknown.

In general, the relative difference between the estimates of the reliability index β resulted within 1% and the relative difference of the corresponding probability of failure P_f resulted within 3%. The difference can be reduced by adopting a smaller value of the tolerance; in the performed tests it was set to $r_{tolerance}$=0.005.

For some choices of the initial values of the variables the computation diverges.

Run ID	Initial Values				Results	
	fy (N/m^2)	L (N/m)	D (N/m)	ξ	r min (beta)	Fail Prob
Status61	345000000	3651.0	11096.0	1.0	3.621603	1.464E-004
Status63	310500000	4016.1	12205.6	1.1	3.621176	1.466E-004
Status62	379500000	3285.9	9986.4	0.9	3.628654	1.425E-004
Status64	396000000	2400.0	10300.0	1.0	3.628783	1.424E-004
Status65	397000000	2399.0	10255.0	1.0	3.619901	1.474E-004

Table 6.2: Initial values of the random variables and corresponding results of five different analyses. All the other parameters of the model are unchanged.

An indication of the relative importance of the random variables is provided by the sensitivity factors α (see section 5.3.1.1.1). It has been shown [50] that if the i-th random variable is replaced with its mean value, then the reliability index β increases by a factor of $1/(1-\alpha_i^2)^{0.5}$.
From the performed calculations (table 6.3) it can be seen that the dead load D could be considered deterministic with little influence on the results.

	Alpha coefficients				Variation of beta by neglecting the random variable			
Run ID	fy	L	D	ξ	fy	L	D	ξ
Status61	-0.5507	0.6740	0.1939	-0.45255	19.81%	35.37%	1.94%	12.14%
Status63	-0.4556	0.7350	0.1493	-0.47954	12.34%	47.47%	1.13%	13.96%
Status62	-0.4557	0.7462	0.2193	-0.43292	12.34%	50.21%	2.50%	10.93%
Status64	-0.4344	0.7648	0.1603	-0.44794	11.02%	55.22%	1.31%	11.85%
Status65	-0.4839	0.7271	0.1676	-0.45728	14.27%	45.65%	1.43%	12.44%
Max	-0.4344	0.7648	0.2193	-0.43292	19.81%	55.22%	2.50%	13.96%
Min	-0.5507	0.6740	0.1493	-0.47954	11.02%	35.37%	1.13%	10.93%

Table 6.3: Sensitivity factors and relative variation of the reliability index β, calculated for some of the performed tests on the simple portal structure.

Calculating the reliability index β of the simple portal structure with the standard FORM procedure required between 200 and 400 iterations, depending on the initial value of the random variables, meaning about 20 to 40 minutes of computation time with the employed hardware.

6.2.1.2 The simplified FORM procedure

The simplified version of the FORM consists in formulating an approximated analytical description of the random variable R (resistance), thus obtaining an analytical formulation of the performance function (6.1), and then solving the minimization problem (5.9) with a gradient method.

The resistance R is taken as a Gaussian random variable.
A deterministic analysis is performed, in which all random variables controlling the resistance are set to their mean value. The resulting Collapse load is taken as the mean value of the resistance μ_R.
A parameter called Central Safety Factor (CSF), which will be useful in the comparison between the two types of FORM, is defined as the ratio between the mean value of the resistance and the mean value of the load:

$$CSF:=\mu_R/(\mu_L+\mu_D) \qquad (6.2)$$

The coefficient of variation of the resistance V_R is taken as that of the most important variable controlling the resistance. In the present case, the only variable is the yield stress f_y of the A992 steel, whose coefficient of variation is $V_{fy}\approx 0.08060$.
Once μ_R and V_R are obtained, they are inserted in a worksheet where the Rackwitz-Fiessler gradient method [41] is implemented (figure 5.10).

Since the analysis requires only one run of the model, computation time was always less than a minute, i.e. approximately 20 to 50 times faster than the standard procedure.

6.2.1.3 Comparison of the two FORM procedures

Several cases were analyzed with both FORM procedures.
The different cases were obtained by varying mean value and standard deviation of the random variables; the geometry of the structure was not altered. Furthermore, some cases were computed several times, starting from different tentative initial values of the random variables.
The results of the analyses are illustrated in figure 6.18.

Run ID	Central Safety Factor	Beta	Failure Probability	Relative Difference (Beta)	Relative Difference (Probability)
Simplified FORM	1.74664	4.52388	3.04E-006		
Status86		4.62130	1.91E-006	0.021	0.592
Simplified FORM	1.74664	3.99146	3.28E-005		
Status82		4.05694	2.49E-005	0.016	0.321
Status84		3.99222	3.27E-005	0.000	0.003
Status85		4.00121	3.15E-005	0.002	0.042
Status66		3.99546	3.23E-005	0.001	0.017
Status68		4.00121	3.15E-005	0.002	0.042
Simplified FORM	1.94081	4.36867	6.25E-006		
Status69		4.37340	6.12E-006	0.001	0.022
Status70		4.37853	5.97E-006	0.002	0.046
Simplified FORM	1.51882	3.36552	3.82E-004		
Status71		3.44551	2.85E-004	0.023	0.340
Status73		3.37318	3.72E-004	0.002	0.028
Simplified FORM	1.74664	3.61934	1.48E-004		
Status61		3.62160	1.46E-004	0.001	0.009
Status63		3.62118	1.47E-004	0.001	0.007
Status62		3.62865	1.42E-004	0.003	0.037
Status64		3.62878	1.42E-004	0.003	0.037
Status65		3.61990	1.47E-004	0.000	0.002
Simplified FORM	1.57197	2.93765	1.65E-003		
Status87		2.97354	1.47E-003	0.012	0.123
Simplified FORM	1.30996	1.95515	2.53E-002		
Status88		1.99477	2.30E-002	0.020	0.098

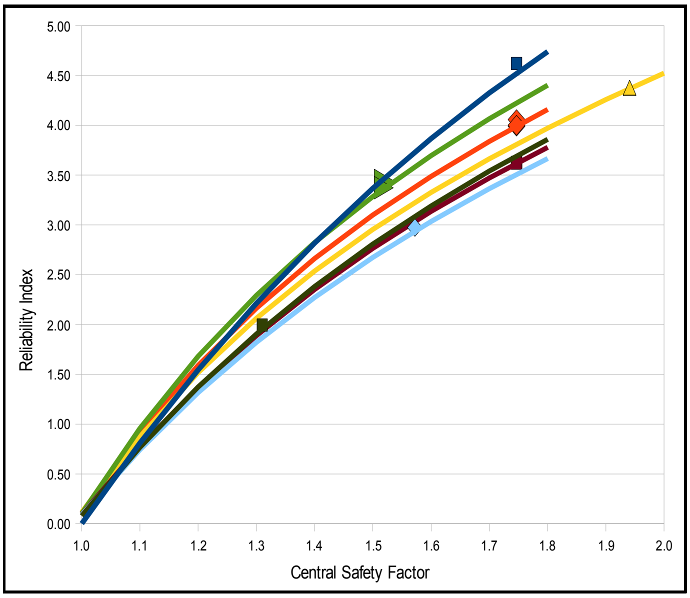


Figure 6.18 : Comparison of the results of several analyses on the simple portal structure. Each color corresponds to a variation of the model. In the Reliability Index/Central Safety Factor diagram, the geometrical shapes represent the results of the standard FORM analyses, while the continuous lines represent the trend of the results of the simplified FORM analyses as the Central Safety Factor varies.

From the table and the diagram in figure 6.18 it can be observed that:
- the results of the two types of FORM follow similar trends;
- the simplified FORM always gives lower values of β than the standard FORM, i.e. the approximation is always on the safe side;
- the maximum relative difference of β calculated with the two procedures always resulted less than 3%;
- the maximum relative difference of the failure probabilities is about 59%.

6.2.2 Three-story structure

The second tested structure has 2x2 bays and 3 stories. Geometry, beam sections and connection types are taken from the three upper stories of the original 5x5 bays, 10 story design developed for NIST, with the exception of the beams of the gravity frames in the north-south direction, which are W14x22 instead of W16x26.

The design includes both gravity and moment-resisting frames.
The beam-column connections are shear-tab type for the gravity frames and WUF-B type for the moment-resisting

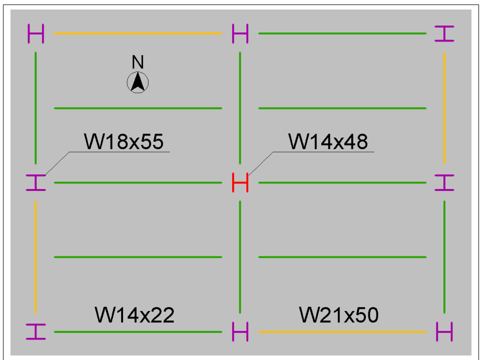


Figure 6.19 : Schematic plan layout of the 2x2x3 structure. Each color identifies an AISC beam shape type. The moment-resisting frames are characterized by the W21x50 beams.

frames, and are modeled with MultiPoint Constraint (MPC) elements as illustrated in figure 6.11.

As with the simple portal, to take into account the geometrical nonlinearities each column is modeled with 5 beam type elements and each beam is modeled with 10 elements.

In all, the model consists of 615 beam type elements and 72 MultiPoint Constraint type elements.

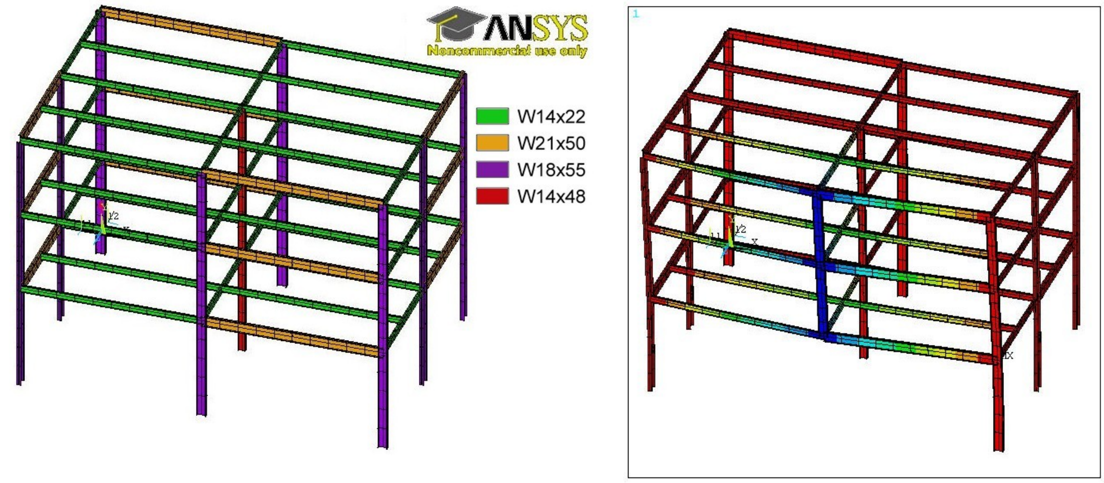


Figure 6.20 : (Left) The used 2x2x3 Finite Element model, with color-coded beam section types. (Right) Deformed shape of the structure at the ultimate limit state after the removal of a column; the colors indicate the vertical displacement.

In the performed analyses, the same performance function (6.1) and the same four random variables (table 6.1) as the simple portal structure are used. In some tests one more variable is included by distinguishing the yielding stress of the steel f_y of the roof floor from that of the other floors. The live load L and the dead load D are vertical and uniformly distributed on the floors.

A series of Structural Vulnerability terms $P(C|LD_{ik})$ of the three-story model were calculated with both FORM procedures.

This time, the parameters of the random variables (mean value and standard deviation) were not altered. Some Local Damage scenarios were computed twice, starting from different tentative initial values of the random variables.

The calculated terms correspond to

- the LD0 case (Damage Level=0, i.e. undamaged structure);
- and 15 LD1 cases (Damage Level=1, i.e. one element at a time is removed). They correspond to 15 scenarios in which one column is removed. The Structural Vulnerability of the 12 remaining LD1 scenarios does not need to be calculated, because of the symmetry of the structure.

The Local Damage scenarios are identified by the element lines nomenclature used in ANSYS, as depicted in figure 6.21.

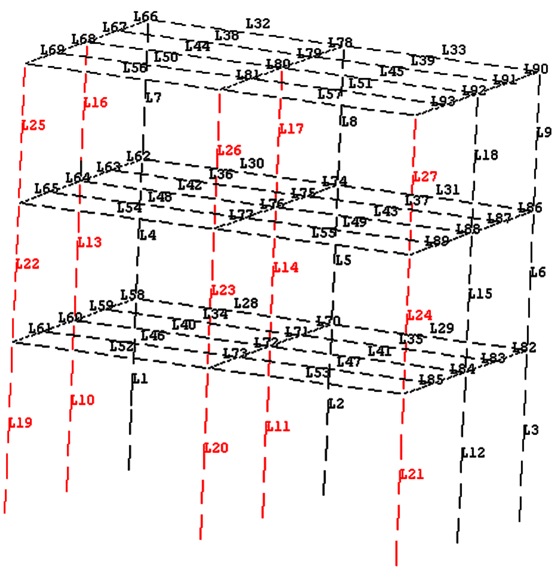


Figure 6.21 : Schematic representation of the model and nomenclature of the element lines. The lines in red were removed, one at a time, to model the LD1 scenarios.

It was observed that both the standard and the simplified FORM algorithms are not able to calculate the probability of failure P_f when this is higher than 50%. This condition approximately corresponds to a value of the Central Safety Factor lower than one or, in other words, to the mean value of the load being higher than the estimated mean value of the resistance. The correspondence would be exact if the model uncertainty term ξ was not present in the definition of the performance function; with the adopted definition (formula 6.1), the probability of failure P_f can be slightly lower than 50% even when the Central Safety Factor is slightly lower than one.
Since with most LD1 scenarios the probability of failure P_f was not computable, to obtain more results a new structure with beam spans scaled to 70% was implemented. The analyzed cases are listed in table 6.4.

First structure – Unmodified geometry

Run ID	Damage (element removed)	Central Safety Factor
3D_IMF_18, 5var_1, 5_var2	Undamaged	2.0261
	Line 19	0.6725
	Line 20	0.3930
	Line 21	0.6113
	Line 11	0.1528
	Line 10	0.6288
	Line 22	0.9257
	Line 23	0.4716
	Line 24	0.7511
	Line 14	0.1659
	Line 13	0.8384
3D_IMF_28	Line 25	1.8864
	Line 26	0.8209
3D_IMF_29	Line 27	1.3624
	Line 17	0.2009
3D_IMF_30	Line 16	1.5894

Second structure – Beam lengths reduced to 70%

Run ID	Damage (element removed)	Central Safety Factor
3D_IMF_27	Undamaged	4.1221
3D_IMF_19, 3D_IMF_20	Line 19	1.5021
	Line 20	0.9781
3D_IMF_31	Line 21	1.4846
	Line 11	0.3144
	Line 10	1.3973
	Line 22	1.9562
3D_IMF_33	Line 23	1.0480
	Line 24	1.6418
	Line 14	0.3406
	Line 13	1.7816
	Line 25	3.8426
	Line 26	1.7117
3D_IMF_32, 5var4, 5var5	Line 27	2.8383
	Line 17	0.4367
3D_IMF_34	Line 16	3.2749

Table 6.4: Summary of the analyses on the three-story structure model. When the Central Safety Factor is lower than 1, the reliability index β and the probability of failure P_f cannot be calculated. In these cases, the cells of the "Run ID" column are empty.

The standard FORM procedure was first implemented on the three-story structure model considering four random variables (yielding stress of steel f_y, live load L, dead load D and uncertainty of the model ξ). In the performed tests, convergence required about 150 to 300 model runs and 2 to 4 hours of computation time.

For some scenarios, analyses were also carried out considering five random variables (yielding stress of steel of the roof floor f_{y1}, yielding stress of steel of the other floors f_{y2}, live load L, dead load D and uncertainty of the model ξ), for which the computation required about 400 to 460 iterations and 4 to 5 hours.

For the simplified FORM procedure the necessary single model run always required less than two minutes of computation time.

6.2.2.1 Comparison of the two FORM procedures

Figure 6.22 reports the results of the analyses.

Run ID	Central Safety Factor	Beta	Failure Probability	Relative Difference (Beta)	Relative Difference (Probability)
Simplified FORM	1.04798	0.43016	3.34E-001		
3D_IMF_33		0.92850	1.77E-001	0.537	0.889
Simplified FORM	1.36238	2.18477	1.45E-002		
3D_IMF_29		2.58976	4.80E-003	0.156	2.010
Simplified FORM	1.48464	2.70474	3.42E-003		
3D_IMF_31		3.04781	1.15E-003	0.113	1.965
Simplified FORM	1.50211	2.77343	2.77E-003		
3D_IMF_19		3.17608	7.46E-004	0.127	2.716
3D_IMF_20		3.18105	7.34E-004	0.128	2.780
Simplified FORM	1.58944	3.09874	9.72E-004		
3D_IMF_30		3.53072	2.07E-004	0.122	3.689
Simplified FORM	1.88637	4.02660	2.83E-005		
3D_IMF_28		4.50283	3.35E-006	0.106	7.439
Simplified FORM	2.02610	4.39325	5.58E-006		
3D_IMF_18		4.47346	3.85E-006	0.018	0.451
Simplified FORM	2.02610	4.39325	5.58E-006		
5var_1		4.50480	3.32E-006	0.025	0.681
5var_2		4.40685	5.24E-006	0.003	0.065
Simplified FORM	2.83828	6.02268	8.58E-010		
5var_4		6.65347	1.43E-011	0.095	58.929
5var_5		6.66239	1.35E-011	0.096	62.678
Simplified FORM	2.83828	6.02268	8.58E-010		
3D_IMF_32		6.97415	1.54E-012	0.136	556.494
Simplified FORM	3.27494	6.68104	1.19E-011		
3D_IMF_34		7.39056	7.31E-014	0.096	161.388
Simplified FORM	4.12206	7.70935	6.33E-015		
3D_IMF_27		9.34445	n/a	0.175	

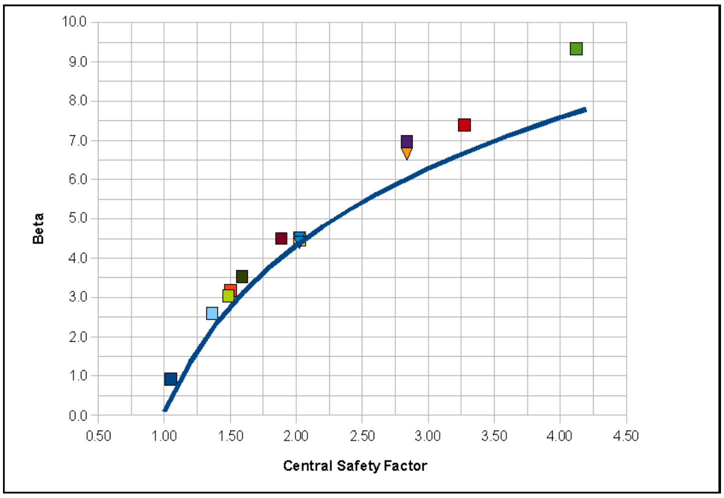
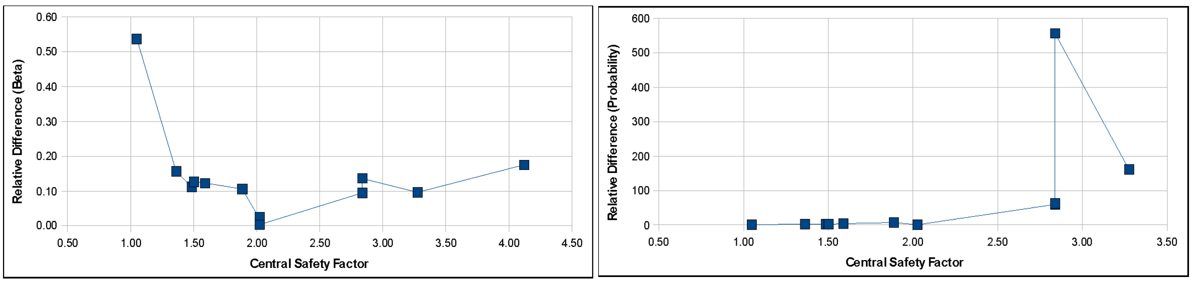


Figure 6.22 : Comparison of the results of the analyses on the three-story structure model. Each color corresponds to a different Local Damage scenario, as listed in table 6.4. In the Reliability Index/Central Safety Factor diagram, the continuous line represents trend of the results of the simplified FORM analyses as the Central Safety Factor varies; each square represents the result of a standard FORM procedure computation with 4 random variables (identified by the label "3D_IMF_"), while the triangles refer to computations with 5 variables (identified by the label "5var_").

From the table and the diagram in figure 6.22 it can be observed that:

- the results of the two types of FORM follow a similar trend;
- the simplified FORM always resulted on the safe side, i.e. the reliability index β is underestimated and the probability of failure P_f is overestimated;

Furthermore, for high values of the Central Safety Factor (CSF) the absolute distance between the two estimates of the reliability index β tends to increase. Yet, the relative difference is relatively steady; as figure 6.23 (left) illustrates, the only value higher than 20% corresponds to the lower value of the CSF.
Since the relationship (5.8) between β and P_f is highly nonlinear, the relative difference between the probabilities of failure gets higher for high CSF values (figure 6.23, center).
In particular, for values of the CSF up to 2.026 the estimates of P_f always resulted in the same order of magnitude, while for higher values of the CSF the difference becomes two or three orders of magnitude.
It must also be observed that no worksheet was able to calculate the failure probability for β=9.344 (case 3D_IMF_27).



Beta	Failure Probability
0.00000	5.00E-001
1.28155	1.00E-001
2.32635	1.00E-002
3.09023	1.00E-003
3.71902	1.00E-004
4.26489	1.00E-005
4.75342	1.00E-006
5.19934	1.00E-007
5.61200	1.00E-008
5.99781	1.00E-009
6.36134	1.00E-010
6.70602	1.00E-011
7.03448	1.00E-012
7.34880	1.00E-013
7.65063	1.00E-014

Figure 6.23 : (Left) Diagram of the relative difference in the estimates of the reliability index β as the Central Safety Factor varies, in the performed tests. (Center) Diagram of the relative difference of the probability of failure P_f as the CSF varies, in the performed tests. (Right) Table of the relationship between β and P_f .

6.2.3 Nine-story structure

The original design of the full structure has 10 stories and 5x5 bays (figure 6.12). Its model was implemented on a version of the software ANSYS that admits an unlimited number of elements and nodes.
Because of limitations of the available software license, for the tests a 9 story, 5x4 bays model was implemented. With reference to figure 6.8, the central span in the north-south direction (between the reference lines 3 and 4) was removed; with reference to figure 6.9, the ground floor was removed, and for the "new" ground floor the 5.33 m height was retained.
The model consists of 7560 beam type elements and 1062 MultiPoint Constraint elements (as opposed to 10300 and 1460, respectively, of the original model). As usual, columns and beams are respectively composed of 5 and 10 elements.
Four random variables were considered (yielding stress of steel f_y, live load L, dead load D and uncertainty of the model ξ).

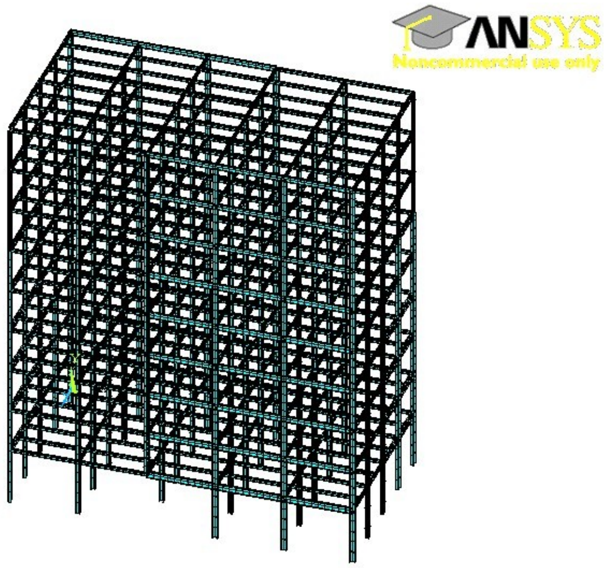


Figure 6.24: The Finite Element model of the nine-story structure, in the analyzed Local Damage scenario (the column on the ground floor, second from left is removed).

Run ID	Performed iterations	Computation time
BigStructure3	97	19h 26min
BigStructure4	99	20h 13min
BigStructure5	135	27h 7min
BigStructure7	106	21h 20min
Total	437	88h 6min

Table 6.5: Summary of the calculation time that was required to perform the standard FORM analysis.

	Central Safety Factor	Beta	Failure Probability Pf
Simplified FORM	2.1658	4.727199	1.14E-006
Standard FORM	2.1658	5.247502	7.71E-008
Relative difference		0.099	13.764

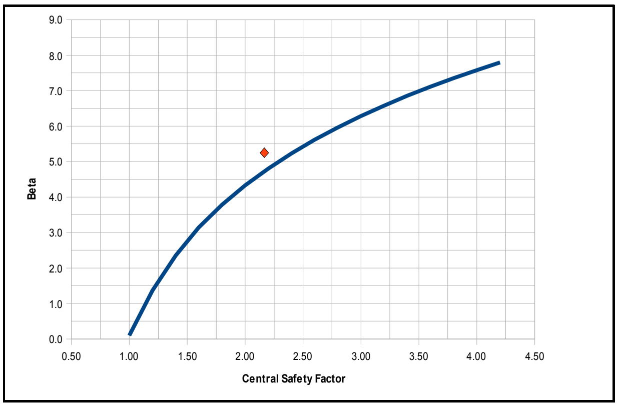


Figure 6.25: Comparison of results of the performed analysis on the nine-story structure model. The square represents the result of the standard FORM analysis; the continuous line represents the trend of the results of the simplified FORM analysis as the Central Safety Factor varies.

One value of the Structural Vulnerability was calculated with the standard FORM procedure. It required four restarts of the algorithm and a total computation time of more than 88 hours, as illustrated in table 6.5. The analysis refers to the Local Damage scenario illustrated in figure 6.24, in which one ground floor column is removed.
The simplified FORM procedure required about 12 minutes of computation time, i.e. about 440 times less than the standard FORM.

The results of the analyses are reported figure 6.25.
Similarly to the other two analyzed structures, the results of the two types of FORM follow a similar trend, and the simplified FORM overestimates the probability of failure.
The relative difference of the reliability index β resulted about 10%, and there are two orders of magnitude of difference in the estimates of the probability of failure P_f.

6.2.4 Structural Vulnerability calculation - Summary

The previous sections presented examples in which Structural Vulnerability terms $P(C|LD_{ik})$ are calculated on structures of increasing size and complexity.
The examples were performed to test and debug the algorithms used for the analyses, to study the feasibility of the proposed approaches, and to find out their problems and limitations.

All performed tests showed that the results obtained with the simplified FORM procedure, which is more approximated, follow the same trends of those obtained with the standard FORM, and always resulted on the “safe” side.
The estimated values of the Structural Vulnerability terms resulted in the same order of magnitude for lower values of the Central Safety Factor, while for higher values the difference resulted 2 to 3 orders of magnitude.
It was observed that both the standard and the simplified FORM algorithms are not able to calculate the probability of failure P_f when this is higher than 50%.
The simplified procedure always resulted much faster to perform than the standard, with the ratio of the required computation time getting bigger as the model size increases (up to 50 times for the simple portal, 150 times for the three-story structure and 440 times for the nine-story structure).
Time constraints prevented from performing more testing, which will be required to further validate the methodology.

6.3 Example of Structural Risk calculation

The first proposed methodology to calculate the Structural Risk (section 5.2.2) was tested on the three-story structure described in section 6.2.2. An incomplete test was also performed on the nine-story structure described in section 6.2.3. The following sections describe these tests.
The target of the tests is the assessment of feasibility and computational effort of the methodology, rather than studying the structure itself. It must be pointed out that the obtained results are not really representative of the structure, because the used characterization of the Local Damage is partially conventional and because some details that might have important effects are not included in the structural model (namely, the dynamic effects and the interaction of the frame with wall panels and floor slabs; section 8.1.1 describes how the dynamic effects can be included in the methodology).

6.3.1 Three-story structure

The Structural Risk of the three-story structure is estimated using equation (5.4). To do it, a characterization of the Local Damage terms $P(LD_{ik})$ is needed, and the Structural Vulnerability terms $P(C|LD_{ik})$ must be calculated.

6.3.1.1 Characterization of the Local Damage terms $P(LD_{ik})$

In this example three Hazards are considered: gas explosion, bomb explosion and vehicle collision. Table 6.6 reports the probability of Damage occurrence because of these Hazards according to Leyendecker and Burnett [23] (section 2.2.1.1).

	P(LD) (/yr, /dwelling unit)
Gas explosion	2.50E-006
Bomb explosion	3.40E-007
Vehicle collision	8.60E-005

Table 6.6: Probabilities of Damage occurrence for the considered Hazards, as reported by Leyendecker and Burnett [23].

	Gas explosion	Bomb and collision
LD1	85%	85%
LD2	8%	10%
LD3	4%	5%
LD4	2%	0%
LD5	1%	0%
Total	100%	100%

Table 6.7: Proportions of the Local Damage levels caused by the considered Hazards.

Since information about the intensity of the Hazards and their consequent direct Damage is not available, the proportions listed in table 6.7 are adopted arbitrarily (i.e, it is assumed that, of all gas explosions, 85% will damage only one structural element, 8% will damage two elements, and so on).

A total of 416 scenarios are considered, with Local Damage level ranging from 0 to 5 (i.e., from undamaged structure to 5 damaged elements). Because of the symmetry of the structure, only 210 scenarios need to be actually computed.

The nomenclature of the beam and column elements of the three-story structure is illustrated in figure 6.26.
Table 6.8 lists the considered Local Damage scenarios and their probabilities. For ease of reading, the scenarios are subdivided in three groups: those in which only columns are damaged (reported in blue in the table), those in which only beams are damaged (yellow), and those mixed (pink).

More in detail, in table 6.8:

- The first column identifies the Local Damage intensity and the floor of the damaged element(s).
- The second column reports the multiplicity of the scenario. If this is 2, then only one term needs to be calculated for two scenarios.
- The third column identifies each scenario with a progressive number.
- The fourth column lists the removed structural element(s) of each scenario, according to the nomenclature of figure 6.26. Some structural elements are identified by two numbers, that are reported within parentheses; for example, finite elements 68 and 69 together make a single structural element.
- The fifth column reports coefficients that take into account different levels of exposure for gas explosions. For each floor, the product of these coefficients with the respective multiplicity sums up to the value 2 (this is because the probabilities of table 6.6 are per dwelling unit, and it is assumed that the structure houses two dwelling units per floor).
- The sixth column reports the probability of occurrence of each scenario due to a gas explosion. Its values are given by the product of $P(LD)_{gas}$ (from table 6.6), proportion coefficient (from table 6.7), multiplicity (second column), relative exposure coefficient (fifth column) and in some cases are divided by 2 or 3. This last division is necessary because the scenarios were subdivided in three groups (blue, yellow and pink). For example the case LD1 at first floor is divided by 2 because it happens in two groups (blue and yellow).
- The seventh column reports the exposure coefficients for bomb explosion and vehicle collision. It is postulated that only elements at the first (ground) floor can be damaged by these Hazards.
- The eighth column reports the probability of occurrence of each scenario due to bomb explosion and vehicle collision. It is calculated similarly to the elements of the sixth column.
- The ninth column reports the total probability of occurrence of each scenario; it is obtained by summing columns six and eight.

From table 6.8 it can be seen that the probability of occurrence P(LD_{ik}) of scenarios 53, 54, 63 and 64 results equal to zero. This is because it was assumed that such Damage scenarios (three columns in a row) can occur only at the ground floor, as a consequence of vehicular impact or bomb explosion.

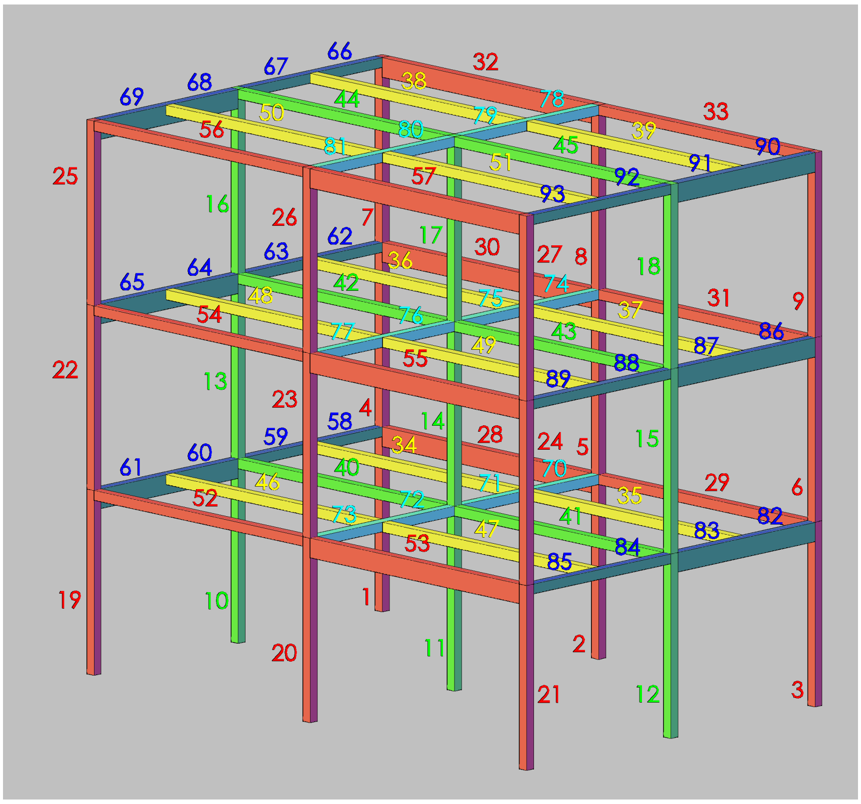


Figure 6.26: Nomenclature of beams and columns of the three-story building model.

Damage Level and Floor	Multiplicity	Scenario ID#	Removed Elements ID#	Relative P(LDik) Gas explosion	P(LDik) gas	Relative P(LDik) Collision+Bomb	P(LDik) collision+bomb	P(LDik)
LD0	1	1	0	0	-	-	-	9.99812E-01
	2	2	19	1/6	3.54E-007	1/4	3.67E-005	3.70E-005
	2	3	20	1/3	7.08E-007	1/4	3.67E-005	3.74E-005
LD1	2	4	21	1/6	3.54E-007	1/4	3.67E-005	3.70E-005
First floor	2	5	10	1/6	3.54E-007	1/4	3.67E-005	3.70E-005
	1	6	11	1/3	3.54E-007	0	0.00E+000	3.54E-007
	2	7	22	1/6	3.54E-007	0	0.00E+000	3.54E-007
	2	8	23	1/3	7.08E-007	0	0.00E+000	7.08E-007
LD1	2	9	24	1/6	3.54E-007	0	0.00E+000	3.54E-007
Second floor	2	10	13	1/6	3.54E-007	0	0.00E+000	3.54E-007
	1	11	14	1/3	3.54E-007	0	0.00E+000	3.54E-007
	2	12	25	1/6	3.54E-007	0	0.00E+000	3.54E-007
	2	13	26	1/3	7.08E-007	0	0.00E+000	7.08E-007
LD1	2	14	27	1/6	3.54E-007	0	0.00E+000	3.54E-007
Third floor	2	15	16	1/6	3.54E-007	0	0.00E+000	3.54E-007
	1	16	17	1/3	3.54E-007	0	0.00E+000	3.54E-007
	2	17	19+20	1/7	1.90E-008	1/4	4.32E-006	4.34E-006
	2	18	20+21	1/7	1.90E-008	1/4	4.32E-006	4.34E-006
LD2	2	19	19+10	1/7	1.90E-008	1/4	4.32E-006	4.34E-006
First floor	2	20	21+12	1/7	1.90E-008	1/4	4.32E-006	4.34E-006
	2	21	20+11	2/7	3.81E-008	0	0.00E+000	3.81E-008
	2	22	11+12	1/7	1.90E-008	0	0.00E+000	1.90E-008
	2	23	22+23	1/7	1.90E-008	0	0.00E+000	1.90E-008
	2	24	23+24	1/7	1.90E-008	0	0.00E+000	1.90E-008
LD2	2	25	22+13	1/7	1.90E-008	0	0.00E+000	1.90E-008
Second floor	2	26	24+15	1/7	1.90E-008	0	0.00E+000	1.90E-008
	2	27	23+14	2/7	3.81E-008	0	0.00E+000	3.81E-008
	2	28	14+15	1/7	1.90E-008	0	0.00E+000	1.90E-008
	2	29	25+26	1/7	1.90E-008	0	0.00E+000	1.90E-008
	2	30	26+27	1/7	1.90E-008	0	0.00E+000	1.90E-008
LD2	2	31	25+16	1/7	1.90E-008	0	0.00E+000	1.90E-008
Third floor	2	32	27+18	1/7	1.90E-008	0	0.00E+000	1.90E-008
	2	33	26+17	2/7	3.81E-008	0	0.00E+000	3.81E-008
	2	34	17+18	1/7	1.90E-008	0	0.00E+000	1.90E-008
	2	35	19+20+10	1/8	8.33E-009	1/4	2.16E-006	2.17E-006
	2	36	19+20+11	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	37	19+10+11	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	38	20+10+11	1/8	8.33E-009	0	0.00E+000	8.33E-009
LD3	2	39	21+11+12	1/8	8.33E-009	0	0.00E+000	8.33E-009
First floor	2	40	21+20+11	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	41	21+20+12	1/8	8.33E-009	1/4	2.16E-006	2.17E-006
	2	42	20+11+12	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	43	19+20+21	0	0.00E+000	1/4	2.16E-006	2.16E-006
	2	44	21+12+3	0	0.00E+000	1/4	2.16E-006	2.16E-006
	2	45	22+23+13	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	46	22+23+14	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	47	22+13+14	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	48	23+13+14	1/8	8.33E-009	0	0.00E+000	8.33E-009
LD3	2	49	24+14+15	1/8	8.33E-009	0	0.00E+000	8.33E-009
Second floor	2	50	24+23+14	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	51	24+23+15	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	52	23+14+15	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	53	22+23+24	0	0.00E+000	0	0.00E+000	0.00E+000
	2	54	24+15+6	0	0.00E+000	0	0.00E+000	0.00E+000
	2	55	25+26+16	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	56	25+26+17	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	57	25+16+17	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	58	26+16+17	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	59	27+17+18	1/8	8.33E-009	0	0.00E+000	8.33E-009
LD3	2	60	27+26+17	1/8	8.33E-009	0	0.00E+000	8.33E-009
Third floor	2	61	27+26+18	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	62	26+17+18	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	63	25+26+27	0	0.00E+000	0	0.00E+000	0.00E+000
	2	64	27+18+9	0	0.00E+000	0	0.00E+000	0.00E+000
LD4	2	65	19+20+10+11	1/2	1.67E-008	0	0.00E+000	1.67E-008
First floor	2	66	20+11+21+12	1/2	1.67E-008	0	0.00E+000	1.67E-008
LD4	2	67	22+23+13+14	1/2	1.67E-008	0	0.00E+000	1.67E-008
Second floor	2	68	23+14+24+15	1/2	1.67E-008	0	0.00E+000	1.67E-008
LD4	2	69	25+26+16+17	1/2	1.67E-008	0	0.00E+000	1.67E-008
Third floor	2	70	26+17+27+18	1/2	1.67E-008	0	0.00E+000	1.67E-008

Table 6.8 (part 1 of 3): Considered Local Damage scenarios and their probabilities. The term $P(LD_{ik})$ for scenario #1 (undamaged structure) is taken as the complementary of the sum of the other terms.

Damage Level and Floor	Multiplicity	Scenario ID#	Removed Elements ID#	Relative exposure Gas explosion	P(LDik) gas	Relative exposure Collision+Bomb	P(LDik) collision+bomb	P(LDik)
LD1 First floor	2	71	52	2/9	4.72E-007	0	0.00E+000	4.72E-007
	2	72	53	1/9	2.36E-007	0	0.00E+000	2.36E-007
	2	73	46	2/9	4.72E-007	0	0.00E+000	4.72E-007
	2	74	47	2/9	4.72E-007	0	0.00E+000	4.72E-007
	2	75	40	2/9	4.72E-007	0	0.00E+000	4.72E-007
LD1 Second floor	2	76	54	2/9	4.72E-007	0	0.00E+000	4.72E-007
	2	77	55	1/9	2.36E-007	0	0.00E+000	2.36E-007
	2	78	48	2/9	4.72E-007	0	0.00E+000	4.72E-007
	2	79	49	2/9	4.72E-007	0	0.00E+000	4.72E-007
	2	80	42	2/9	4.72E-007	0	0.00E+000	4.72E-007
LD1 Third floor	2	81	56	2/9	4.72E-007	0	0.00E+000	4.72E-007
	2	82	57	1/9	2.36E-007	0	0.00E+000	2.36E-007
	2	83	50	2/9	4.72E-007	0	0.00E+000	4.72E-007
	2	84	51	2/9	4.72E-007	0	0.00E+000	4.72E-007
	2	85	44	2/9	4.72E-007	0	0.00E+000	4.72E-007
LD2 First floor	2	86	(60+61)+46	1/3	4.44E-008	0	0.00E+000	4.44E-008
	2	87	(84+85)+47	2/3	8.89E-008	0	0.00E+000	8.89E-008
LD2 Second floor	2	88	(64+65)+48	1/3	4.44E-008	0	0.00E+000	4.44E-008
	2	89	(88+89)+49	2/3	8.89E-008	0	0.00E+000	8.89E-008
LD2 Third floor	2	90	(68+69)+50	1/3	4.44E-008	0	0.00E+000	4.44E-008
	2	91	(92+93)+51	2/3	8.89E-008	0	0.00E+000	8.89E-008
LD3 First floor	2	92	(72+73)+46+47	1	6.67E-008	0	0.00E+000	6.67E-008
LD3 Second floor	2	93	(76+77)+48+49	1	6.67E-008	0	0.00E+000	6.67E-008
LD3 Third floor	2	94	(80+81)+50+51	1	6.67E-008	0	0.00E+000	6.67E-008
LD4 First floor	2	95	(60+61)+46+52+40	1/2	1.67E-008	0	0.00E+000	1.67E-008
	2	96	(84+85)+47+53+41	1/2	1.67E-008	0	0.00E+000	1.67E-008
LD4 Second floor	2	97	(64+65)+48+54+42	1/2	1.67E-008	0	0.00E+000	1.67E-008
	2	98	(88+89)+49+55+43	1/2	1.67E-008	0	0.00E+000	1.67E-008
LD4 Third floor	2	99	(68+69)+50+56+44	1/2	1.67E-008	0	0.00E+000	1.67E-008
	2	100	(92+93)+51+57+45	1/2	1.67E-008	0	0.00E+000	1.67E-008
LD2 First floor	2	101	19+52	1/6	2.22E-008	0	0.00E+000	2.22E-008
	2	102	21+53	1/6	2.22E-008	0	0.00E+000	2.22E-008
	2	103	20+52	1/6	2.22E-008	0	0.00E+000	2.22E-008
	2	104	20+53	1/6	2.22E-008	0	0.00E+000	2.22E-008
	2	105	12+41	1/6	2.22E-008	0	0.00E+000	2.22E-008
	2	106	11+41	1/6	2.22E-008	0	0.00E+000	2.22E-008
	2	107	22+52	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	108	24+53	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	109	23+52	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	110	23+53	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	111	15+41	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	112	14+41	1/9	1.11E-008	0	0.00E+000	1.11E-008
LD2 Second floor	2	113	22+54	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	114	24+55	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	115	23+54	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	116	23+55	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	117	15+43	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	118	14+43	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	119	25+54	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	120	27+55	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	121	26+54	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	122	26+55	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	123	18+43	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	124	17+43	1/9	1.11E-008	0	0.00E+000	1.11E-008
LD2 Third floor	2	125	25+56	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	126	27+57	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	127	26+56	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	128	26+57	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	129	18+45	1/9	1.11E-008	0	0.00E+000	1.11E-008
	2	130	17+45	1/9	1.11E-008	0	0.00E+000	1.11E-008
LD3 First floor	2	131	19+52+20	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	132	20+53+21	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	133	11+41+12	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	134	22+52+23	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	135	23+53+24	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	136	14+41+15	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	137	19+(60+61)+46	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	138	21+(84+85)+47	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	139	10+(60+61)+46	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	140	12+(84+85)+47	1/8	8.33E-009	0	0.00E+000	8.33E-009
	2	141	22+60+61	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	142	24+84+85	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	143	13+60+61	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	144	15+84+85	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	145	20+52+53	1/8	8.33E-009	0	0.00E+000	8.33E-009

Table 6.8 (part 2 of 3): Considered Local Damage scenarios and their probabilities.

Damage Level and Floor	Multiplicity	Scenario ID#	Removed Elements ID#	Relative P(LDik) Gas explosion	P(LDik) gas	Relative P(LDik) Collision+Bomb	P(LDik) collision+bomb	P(LDik)
	2	147	22+54+23	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	148	23+55+24	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	149	14+43+15	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	150	25+54+26	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	151	26+55+27	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	152	17+43+18	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	153	22+(64+65)+48	1/9	4.17E-009	0	0.00E+000	4.17E-009
LD3	2	154	24+(88+89)+49	1/9	4.17E-009	0	0.00E+000	4.17E-009
Second floor	2	155	13+(64+65)+48	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	156	15+(88+89)+49	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	157	25+(64+65)+48	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	158	27+(88+89)+49	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	159	16+(64+65)+48	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	160	18+(88+89)+49	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	161	23+54+55	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	162	26+54+55	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	163	25+56+26	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	164	26+57+27	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	165	17+45+18	1/9	4.17E-009	0	0.00E+000	4.17E-009
LD3	2	166	25+(68+69)+50	1/9	4.17E-009	0	0.00E+000	4.17E-009
Third floor	2	167	27+(92+93)+51	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	168	16+(68+69)+50	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	169	18+(92+93)+51	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	170	26+56+57	1/9	4.17E-009	0	0.00E+000	4.17E-009
	2	171	19+10+(60+61)+46	1/4	8.33E-009	0	0.00E+000	8.33E-009
	2	172	19+52+(60+61)+46	1/4	8.33E-009	0	0.00E+000	8.33E-009
	2	173	21+12+(84+85)+47	1/4	8.33E-009	0	0.00E+000	8.33E-009
LD4	2	174	21+53+(84+85)+47	1/4	8.33E-009	0	0.00E+000	8.33E-009
First floor	2	175	22+13+(60+61)+46	1/8	4.17E-009	0	0.00E+000	4.17E-009
	2	176	22+52+(60+61)+46	1/8	4.17E-009	0	0.00E+000	4.17E-009
	2	177	24+15+(84+85)+47	1/8	4.17E-009	0	0.00E+000	4.17E-009
	2	178	24+53+(84+85)+47	1/8	4.17E-009	0	0.00E+000	4.17E-009
	2	179	22+13+(64+65)+48	1/8	4.17E-009	0	0.00E+000	4.17E-009
	2	180	22+54+(64+65)+48	1/8	4.17E-009	0	0.00E+000	4.17E-009
	2	181	24+15+(88+89)+49	1/8	4.17E-009	0	0.00E+000	4.17E-009
LD4	2	182	24+55+(88+89)+49	1/8	4.17E-009	0	0.00E+000	4.17E-009
Second floor	2	183	25+16+(64+65)+48	1/8	4.17E-009	0	0.00E+000	4.17E-009
	2	184	25+54+(64+65)+48	1/8	4.17E-009	0	0.00E+000	4.17E-009
	2	185	27+18+(88+89)+49	1/8	4.17E-009	0	0.00E+000	4.17E-009
	2	186	27+55+(88+89)+49	1/8	4.17E-009	0	0.00E+000	4.17E-009
	2	187	25+16+(68+69)+50	1/8	4.17E-009	0	0.00E+000	4.17E-009
LD4	2	188	25+56+(68+69)+50	1/8	4.17E-009	0	0.00E+000	4.17E-009
Third floor	2	189	27+18+(92+93)+51	1/8	4.17E-009	0	0.00E+000	4.17E-009
	2	190	27+57+(92+93)+51	1/8	4.17E-009	0	0.00E+000	4.17E-009
	2	191	19+10+(60+61)+46+52	1/4	1.25E-008	0	0.00E+000	1.25E-008
	2	192	19+20+(60+61)+46+52	1/4	1.25E-008	0	0.00E+000	1.25E-008
	2	193	21+12+(84+85)+47+53	1/4	1.25E-008	0	0.00E+000	1.25E-008
LD5	2	194	21+20+(84+85)+47+53	1/4	1.25E-008	0	0.00E+000	1.25E-008
First floor	2	195	22+13+(60+61)+46+52	1/8	6.25E-009	0	0.00E+000	6.25E-009
	2	196	22+23+(60+61)+46+52	1/8	6.25E-009	0	0.00E+000	6.25E-009
	2	197	24+15+(84+85)+47+53	1/8	6.25E-009	0	0.00E+000	6.25E-009
	2	198	24+23+(84+85)+47+53	1/8	6.25E-009	0	0.00E+000	6.25E-009
	2	199	22+13+(64+65)+48+54	1/8	6.25E-009	0	0.00E+000	6.25E-009
	2	200	22+23+(64+65)+48+54	1/8	6.25E-009	0	0.00E+000	6.25E-009
	2	201	24+15+(88+89)+49+55	1/8	6.25E-009	0	0.00E+000	6.25E-009
LD5	2	202	24+23+(88+89)+49+55	1/8	6.25E-009	0	0.00E+000	6.25E-009
Second floor	2	203	25+16+(64+65)+48+54	1/8	6.25E-009	0	0.00E+000	6.25E-009
	2	204	25+26+(64+65)+48+54	1/8	6.25E-009	0	0.00E+000	6.25E-009
	2	205	27+18+(88+89)+49+55	1/8	6.25E-009	0	0.00E+000	6.25E-009
	2	206	27+26+(88+89)+49+55	1/8	6.25E-009	0	0.00E+000	6.25E-009
	2	207	25+16+(68+69)+50+56	1/8	6.25E-009	0	0.00E+000	6.25E-009
LD5	2	208	25+26+(68+69)+50+56	1/8	6.25E-009	0	0.00E+000	6.25E-009
Third floor	2	209	27+18+(92+93)+51+57	1/8	6.25E-009	0	0.00E+000	6.25E-009
	2	210	27+26+(92+93)+51+57	1/8	6.25E-009	0	0.00E+000	6.25E-009
Sum=	416				1.5000E-005		1.7268E-004	1.8768E-004

Table 6.8 (part 3 of 3): Considered Local Damage scenarios and their probabilities.

6.3.1.2 Calculation of the Structural Vulnerability terms $P(C|LD_{ik})$

The Structural Vulnerability terms $P(C|LD_{ik})$ are calculated with the simplified FORM procedure and static non linear analysis, as in the examples of section 6.2.

An algorithm was implemented in ANSYS to automatically calculate the estimates of the resistance R in the 210 considered Local Damage scenarios. Then the $P(C|LD_{ik})$ terms are obtained by inserting the values of R in the implemented Rackwitz-Fiessler algorithm worksheet. Since the algorithm is not able to calculate the probability of failure P_f when this is higher than 50%, in these cases the $P(C\mid LD_{ik})$ terms are conservatively assumed equal to 1.

The parameters of the calculations are reported in table 6.9, in which:

- the first column identifies the scenario ID number, as defined in table 6.8;
- the second column reports the calculated values of the resistance R;
- the third column reports the values of the Central Safety Factor;
- the fourth column reports the values of the reliability index β;
- the fifth column reports the calculated Structural Vulnerability terms;
- the sixth column reports the probability of occurrence of each considered Local Damage scenario, taken from table 6.8;
- the seventh column reports the contributions of each scenario to the Structural Risk.

It must be noticed that, in the scenarios in which only beams were removed, the resistance resulted the same as in the undamaged structure. This is because no horizontal loads or column eccentricities were included in the model.

Scenario ID#	Resistance (N/m)	CSF	β	$P(C\mid LD_{ik})$	$P(LD_{ik})$	$P(C_{ik}=1)$
1	34'774	4.12	7.71	6.33E-015	9.99899E-01	6.33E-015
2	12'672	1.50	2.77	2.77E-003	3.70E-005	1.03E-007
3	8'251	0.98	0.07	4.72E-001	3.74E-005	1.77E-005
4	12'524	1.48	2.70	3.42E-003	3.70E-005	1.27E-007
5	11'788	1.40	2.34	9.62E-003	3.70E-005	3.56E-007
6	2'652	0.31	n/a	1.00E+000	3.54E-007	3.54E-007
7	16'503	1.96	4.21	1.25E-005	3.54E-007	4.43E-012
8	8'988	1.07	0.55	2.92E-001	7.08E-007	2.07E-007
9	13'998	1.66	3.34	4.20E-004	3.54E-007	1.49E-010
10	15'029	1.78	3.73	9.75E-005	3.54E-007	3.45E-011
11	2'800	0.33	n/a	1.00E+000	3.54E-007	3.54E-007
12	32'416	3.84	7.40	6.84E-014	3.54E-007	2.42E-020
13	14'587	1.73	3.57	1.82E-004	7.08E-007	1.29E-010
14	23'870	2.83	6.01	9.37E-010	3.54E-007	3.32E-016
15	27'406	3.25	6.64	1.52E-011	3.54E-007	5.39E-018
16	3'684	0.44	n/a	1.00E+000	3.54E-007	3.54E-007
17	8'546	1.01	0.19	4.26E-001	4.34E-006	1.85E-006
18	1'473	0.17	n/a	1.00E+000	4.34E-006	4.34E-006
19	2'063	0.24	n/a	1.00E+000	4.34E-006	4.34E-006
20	11'788	1.40	2.34	9.62E-003	4.34E-006	4.17E-008
21	2'505	0.30	n/a	1.00E+000	3.81E-008	3.81E-008
22	2'505	0.30	n/a	1.00E+000	1.90E-008	1.90E-008
23	9'135	1.08	0.66	2.54E-001	1.90E-008	4.83E-009
24	1'621	0.19	n/a	1.00E+000	1.90E-008	1.90E-008
25	2'210	0.26	n/a	1.00E+000	1.90E-008	1.90E-008
26	12'819	1.52	2.84	2.25E-003	1.90E-008	4.28E-011
27	2'800	0.33	n/a	1.00E+000	3.81E-008	3.81E-008
28	2'800	0.33	n/a	1.00E+000	1.90E-008	1.90E-008
29	14'145	1.68	3.40	3.40E-004	1.90E-008	6.48E-012
30	1'326	0.16	n/a	1.00E+000	1.90E-008	1.90E-008
31	1'916	0.23	n/a	1.00E+000	1.90E-008	1.90E-008
32	22'397	2.65	5.71	5.66E-009	1.90E-008	1.08E-016
33	3'389	0.40	n/a	1.00E+000	3.81E-008	3.81E-008
34	3'536	0.42	n/a	1.00E+000	1.90E-008	1.90E-008
35	1'916	0.23	n/a	1.00E+000	2.17E-006	2.17E-006
36	2'652	0.31	n/a	1.00E+000	8.33E-009	8.33E-009
37	1'473	0.17	n/a	1.00E+000	8.33E-009	8.33E-009
38	2'358	0.28	n/a	1.00E+000	8.33E-009	8.33E-009
39	2'505	0.30	n/a	1.00E+000	8.33E-009	8.33E-009
40	1'326	0.16	n/a	1.00E+000	8.33E-009	8.33E-009
41	1'473	0.17	n/a	1.00E+000	2.17E-006	2.17E-006
42	2'652	0.31	n/a	1.00E+000	8.33E-009	8.33E-009
43	1'473	0.17	n/a	1.00E+000	2.16E-006	2.16E-006
44	1'916	0.23	n/a	1.00E+000	2.16E-006	2.16E-006
45	2'063	0.24	n/a	1.00E+000	8.33E-009	8.33E-009
46	2'800	0.33	n/a	1.00E+000	8.33E-009	8.33E-009
47	1'621	0.19	n/a	1.00E+000	8.33E-009	8.33E-009
48	2'652	0.31	n/a	1.00E+000	8.33E-009	8.33E-009
49	2'800	0.33	n/a	1.00E+000	8.33E-009	8.33E-009
50	1'326	0.16	n/a	1.00E+000	8.33E-009	8.33E-009
51	1'621	0.19	n/a	1.00E+000	8.33E-009	8.33E-009
52	2'800	0.33	n/a	1.00E+000	8.33E-009	8.33E-009
53	1'473	0.17	n/a	1.00E+000	0.00E+000	0.00E+000
54	1'916	0.23	n/a	1.00E+000	0.00E+000	0.00E+000
55	2'063	0.24	n/a	1.00E+000	8.33E-009	8.33E-009
56	3'389	0.40	n/a	1.00E+000	8.33E-009	8.33E-009
57	1'473	0.17	n/a	1.00E+000	8.33E-009	8.33E-009
58	3'242	0.38	n/a	1.00E+000	8.33E-009	8.33E-009
59	3'389	0.40	n/a	1.00E+000	8.33E-009	8.33E-009
60	1'031	0.12	n/a	1.00E+000	8.33E-009	8.33E-009
61	1'326	0.16	n/a	1.00E+000	8.33E-009	8.33E-009
62	3'242	0.38	n/a	1.00E+000	8.33E-009	8.33E-009
63	1'179	0.14	n/a	1.00E+000	0.00E+000	0.00E+000
64	n/a	0.00	n/a	1.00E+000	0.00E+000	0.00E+000
65	1'179	0.14	n/a	1.00E+000	1.67E-008	1.67E-008
66	1'326	0.16	n/a	1.00E+000	1.67E-008	1.67E-008
67	1'326	0.16	n/a	1.00E+000	1.67E-008	1.67E-008
68	1'473	0.17	n/a	1.00E+000	1.67E-008	1.67E-008
69	1'473	0.17	n/a	1.00E+000	1.67E-008	1.67E-008
70	1'031	0.12	n/a	1.00E+000	1.67E-008	1.67E-008
71	34'774	4.12	7.71	6.33E-015	4.72E-007	2.99E-021
72	34'774	4.12	7.71	6.33E-015	2.36E-007	1.49E-021
73	34'774	4.12	7.71	6.33E-015	4.72E-007	2.99E-021
74	34'774	4.12	7.71	6.33E-015	4.72E-007	2.99E-021
75	34'774	4.12	7.71	6.33E-015	4.72E-007	2.99E-021
76	34'774	4.12	7.71	6.33E-015	4.72E-007	2.99E-021
77	34'774	4.12	7.71	6.33E-015	2.36E-007	1.49E-021
78	34'774	4.12	7.71	6.33E-015	4.72E-007	2.99E-021
79	34'774	4.12	7.71	6.33E-015	4.72E-007	2.99E-021
80	34'774	4.12	7.71	6.33E-015	4.72E-007	2.99E-021
81	34'774	4.12	7.71	6.33E-015	4.72E-007	2.99E-021
82	34'774	4.12	7.71	6.33E-015	2.36E-007	1.49E-021
83	34'774	4.12	7.71	6.33E-015	4.72E-007	2.99E-021
84	34'774	4.12	7.71	6.33E-015	4.72E-007	2.99E-021
85	34'774	4.12	7.71	6.33E-015	4.72E-007	2.99E-021
86	34'774	4.12	7.71	6.33E-015	4.44E-008	2.81E-022
87	34'774	4.12	7.71	6.33E-015	8.89E-008	5.63E-022
88	34'774	4.12	7.71	6.33E-015	4.44E-008	2.81E-022
89	34'774	4.12	7.71	6.33E-015	8.89E-008	5.63E-022
90	34'774	4.12	7.71	6.33E-015	4.44E-008	2.81E-022
91	34'774	4.12	7.71	6.33E-015	8.89E-008	5.63E-022
92	34'774	4.12	7.71	6.33E-015	6.67E-008	4.22E-022
93	34'774	4.12	7.71	6.33E-015	6.67E-008	4.22E-022
94	34'774	4.12	7.71	6.33E-015	6.67E-008	4.22E-022
95	34'774	4.12	7.71	6.33E-015	1.67E-008	1.05E-022
96	34'774	4.12	7.71	6.33E-015	1.67E-008	1.05E-022
97	34'774	4.12	7.71	6.33E-015	1.67E-008	1.05E-022
98	34'774	4.12	7.71	6.33E-015	1.67E-008	1.05E-022
99	34'774	4.12	7.71	6.33E-015	1.67E-008	1.05E-022
100	34'774	4.12	7.71	6.33E-015	1.67E-008	1.05E-022
101	15'324	1.82	3.83	6.44E-005	2.22E-008	1.43E-012
102	13'556	1.61	3.16	7.88E-004	2.22E-008	1.75E-011
103	8'841	1.05	0.43	3.34E-001	2.22E-008	7.41E-009
104	8'693	1.03	0.31	3.78E-001	2.22E-008	8.41E-009
105	13'851	1.64	3.28	5.18E-004	2.22E-008	1.15E-011
106	2'652	0.31	n/a	1.00E+000	2.22E-008	2.22E-008

Table 6.9 (part 1 of 2): Parameters of the Structural Risk calculation.

Scenario ID#	Resistance (N/m)	CSF	β	P(C|LD$_{ik}$)	P(LD$_{ik}$)	P(C$_{ik}$=1)
107	16'797	1.99	4.30	8.35E-006	1.11E-008	9.28E-014
108	15'029	1.78	3.73	9.75E-005	1.11E-008	1.08E-012
109	8'988	1.07	0.55	2.92E-001	1.11E-008	3.24E-009
110	9'430	1.12	0.88	1.88E-001	1.11E-008	2.09E-009
111	15'029	1.78	3.73	9.75E-005	1.11E-008	1.08E-012
112	2'800	0.33	n/a	1.00E+000	1.11E-008	1.11E-008
113	23'575	2.79	5.95	1.34E-009	1.11E-008	1.49E-017
114	19'450	2.31	5.03	2.40E-007	1.11E-008	2.67E-015
115	10'314	1.22	1.49	6.80E-002	1.11E-008	7.55E-010
116	10'314	1.22	1.49	6.80E-002	1.11E-008	7.55E-010
117	19'450	2.31	5.03	2.40E-007	1.11E-008	2.67E-015
118	3'094	0.37	n/a	1.00E+000	1.11E-008	1.11E-008
119	32'416	3.84	7.40	6.84E-014	1.11E-008	7.60E-022
120	24'165	2.86	6.07	6.58E-010	1.11E-008	7.31E-018
121	14'440	1.71	3.51	2.24E-004	1.11E-008	2.49E-012
122	17'387	2.06	4.48	3.74E-006	1.11E-008	4.16E-014
123	27'406	3.25	6.64	1.52E-011	1.11E-008	1.69E-019
124	3'684	0.44	n/a	1.00E+000	1.11E-008	1.11E-008
125	34'774	4.12	7.71	6.33E-015	1.11E-008	7.03E-023
126	4'715	0.56	n/a	1.00E+000	1.11E-008	1.11E-008
127	19'450	2.31	5.03	2.40E-007	1.11E-008	2.67E-015
128	2'210	0.26	n/a	1.00E+000	1.11E-008	1.11E-008
129	34'774	4.12	7.71	6.33E-015	1.11E-008	7.03E-023
130	3'684	0.44	n/a	1.00E+000	1.11E-008	1.11E-008
131	8'988	1.07	0.55	2.92E-001	8.33E-009	2.43E-009
132	1'768	0.21	n/a	1.00E+000	8.33E-009	8.33E-009
133	2'652	0.31	n/a	1.00E+000	8.33E-009	8.33E-009
134	9'135	1.08	0.66	2.54E-001	4.17E-009	1.06E-009
135	1'621	0.19	n/a	1.00E+000	4.17E-009	4.17E-009
136	2'800	0.33	n/a	1.00E+000	4.17E-009	4.17E-009
137	13'261	1.57	3.04	1.20E-003	8.33E-009	9.99E-012
138	13'556	1.61	3.16	7.88E-004	8.33E-009	6.56E-012
139	12'377	1.47	2.63	4.21E-003	8.33E-009	3.51E-011
140	12'230	1.45	2.56	5.18E-003	8.33E-009	4.32E-011
141	16'797	1.99	4.30	8.35E-006	4.17E-009	3.48E-014
142	13'998	1.66	3.34	4.20E-004	4.17E-009	1.75E-012
143	15'029	1.78	3.73	9.75E-005	4.17E-009	4.06E-013
144	15'029	1.78	3.73	9.75E-005	4.17E-009	4.06E-013
145	8'546	1.01	0.19	4.26E-001	8.33E-009	3.55E-009
146	9'430	1.12	0.88	1.88E-001	4.17E-009	7.84E-010
147	10'462	1.24	1.58	5.65E-002	4.17E-009	2.36E-010
148	2'210	0.26	n/a	1.00E+000	4.17E-009	4.17E-009
149	3'242	0.38	n/a	1.00E+000	4.17E-009	4.17E-009
150	14'145	1.68	3.40	3.40E-004	4.17E-009	1.42E-012
151	1'326	0.16	n/a	1.00E+000	4.17E-009	4.17E-009
152	3'536	0.42	n/a	1.00E+000	4.17E-009	4.17E-009
153	17'682	2.10	4.56	2.51E-006	4.17E-009	1.05E-014
154	15'619	1.85	3.93	4.27E-005	4.17E-009	1.78E-013
155	16'208	1.92	4.12	1.88E-005	4.17E-009	7.83E-014
156	18'860	2.24	4.88	5.20E-007	4.17E-009	2.17E-015
157	30'648	3.63	7.15	4.36E-013	4.17E-009	1.82E-021
158	23'870	2.83	6.01	9.37E-010	4.17E-009	3.91E-018
159	26'522	3.14	6.49	4.16E-011	4.17E-009	1.73E-019

Scenario ID#	Resistance (N/m)	CSF	β	P(C|LD$_{ik}$)	P(LD$_{ik}$)	P(C$_{ik}$=1)
160	27'112	3.21	6.60	2.12E-011	4.17E-009	8.85E-020
161	12'966	1.54	2.91	1.82E-003	4.17E-009	7.60E-012
162	14'440	1.71	3.51	2.24E-004	4.17E-009	9.34E-013
163	19'155	2.27	4.96	3.53E-007	4.17E-009	1.47E-015
164	2'210	0.26	n/a	1.00E+000	4.17E-009	4.17E-009
165	3'684	0.44	n/a	1.00E+000	4.17E-009	4.17E-009
166	1'179	0.14	n/a	1.00E+000	4.17E-009	4.17E-009
167	31'827	3.77	7.32	1.26E-013	4.17E-009	5.26E-022
168	2'505	0.30	n/a	1.00E+000	4.17E-009	4.17E-009
169	31'237	3.70	7.23	2.34E-013	4.17E-009	9.74E-022
170	3'242	0.38	n/a	1.00E+000	4.17E-009	4.17E-009
171	2'210	0.26	n/a	1.00E+000	8.33E-009	8.33E-009
172	16'503	1.96	4.21	1.25E-005	8.33E-009	1.04E-013
173	13'114	1.55	2.97	1.48E-003	8.33E-009	1.23E-011
174	14'735	1.75	3.62	1.48E-004	8.33E-009	1.23E-012
175	2'210	0.26	n/a	1.00E+000	4.17E-009	4.17E-009
176	16'797	1.99	4.30	8.35E-006	4.17E-009	3.48E-014
177	12'819	1.52	2.84	2.25E-003	4.17E-009	9.37E-012
178	15'029	1.78	3.73	9.75E-005	4.17E-009	4.06E-013
179	2'505	0.30	n/a	1.00E+000	4.17E-009	4.17E-009
180	29'175	3.46	6.93	2.14E-012	4.17E-009	8.90E-021
181	15'324	1.82	3.83	6.44E-005	4.17E-009	2.68E-013
182	22'986	2.72	5.83	2.74E-009	4.17E-009	1.14E-017
183	1'916	0.23	n/a	1.00E+000	4.17E-009	4.17E-009
184	30'648	3.63	7.15	4.36E-013	4.17E-009	1.82E-021
185	22'397	2.65	5.71	5.66E-009	4.17E-009	2.36E-017
186	24'165	2.86	6.07	6.58E-010	4.17E-009	2.74E-018
187	1'179	0.14	n/a	1.00E+000	4.17E-009	4.17E-009
188	34'774	4.12	7.71	6.33E-015	4.17E-009	2.64E-023
189	31'237	3.70	7.23	2.34E-013	4.17E-009	9.74E-022
190	34'774	4.12	7.71	6.33E-015	4.17E-009	2.64E-023
191	2'210	0.26	n/a	1.00E+000	1.25E-008	1.25E-008
192	9'578	1.14	0.99	1.61E-001	1.25E-008	2.01E-009
193	12'672	1.50	2.77	2.77E-003	1.25E-008	3.47E-011
194	1'768	0.21	n/a	1.00E+000	1.25E-008	1.25E-008
195	2'210	0.26	n/a	1.00E+000	6.25E-009	6.25E-009
196	9'135	1.08	0.66	2.54E-001	6.25E-009	1.59E-009
197	13'556	1.61	3.16	7.88E-004	6.25E-009	4.92E-012
198	1'621	0.19	n/a	1.00E+000	6.25E-009	6.25E-009
199	n/a	0.00	n/a	1.00E+000	6.25E-009	6.25E-009
200	11'128	1.32	1.98	2.37E-002	6.25E-009	1.48E-010
201	18'860	2.24	4.88	5.20E-007	6.25E-009	3.25E-015
202	n/a	0.00	n/a	1.00E+000	6.25E-009	6.25E-009
203	1'916	0.23	n/a	1.00E+000	6.25E-009	6.25E-009
204	14'145	1.68	3.40	3.40E-004	6.25E-009	2.13E-012
205	23'575	2.79	5.95	1.34E-009	6.25E-009	8.36E-018
206	1'326	0.16	n/a	1.00E+000	6.25E-009	6.25E-009
207	2'505	0.30	n/a	1.00E+000	6.25E-009	6.25E-009
208	24'165	2.86	6.07	6.58E-010	6.25E-009	4.11E-018
209	31'237	3.70	7.23	2.34E-013	6.25E-009	1.46E-021
210	2'210	0.26	n/a	1.00E+000	6.25E-009	6.25E-009
					Sum=	3.95E-005

Table 6.9 (part 2 of 2): Parameters of the Structural Risk calculation.

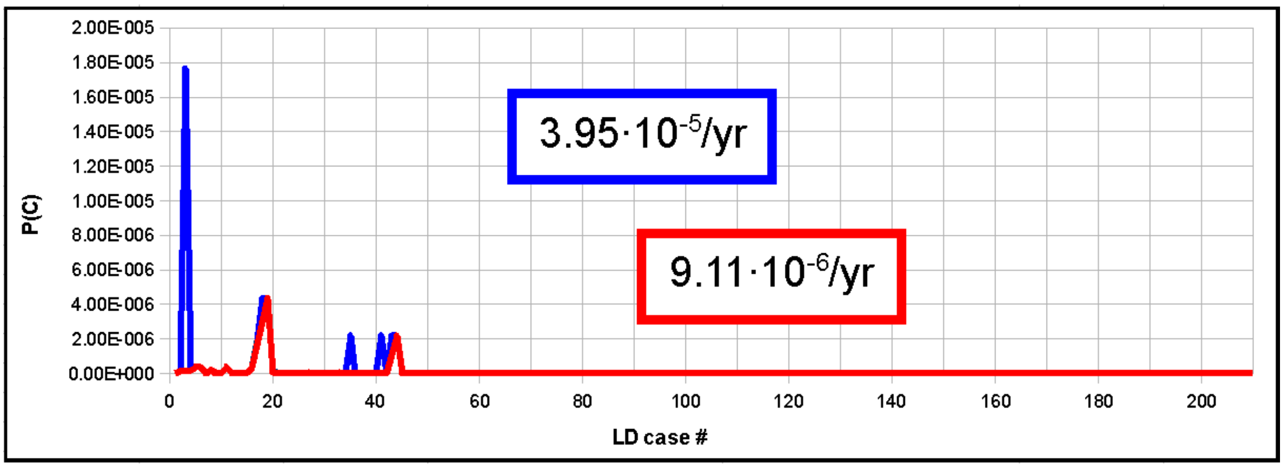


Figure 6.27: Diagram of the 210 calculated values of the P(C_{ik}=1) terms. Blue line: original structure. Red line: modified structure.

The total Structural Risk results P(C=1)= $3.95\cdot10^{-5}$/yr.
This value is higher than the assumed target value $P(C=1)_{acc}=10^{-7}$/yr, thus the analyzed structure would be deemed not acceptable.

The blue line in figure 6.27 illustrates all the 210 values of the $P(C_{ik}=1)$ terms and gives useful information for mitigation measures.

It can be seen that there are several peaks, the highest of which corresponds to scenario #3 (removal of element 20). Several other peaks correspond to scenarios in which element 20 is removed. By applying the Event Control strategy or the Specific Load Resistance strategy the probability of occurrence $P(LD_{ik})$ of these scenarios can be lowered.

The red line in figure 6.27 represents the diagram of $P(C_{ik}=1)$ if element 20 (and its symmetrical element 2) are made less prone to suffer a direct Damage. For the sake of the example, the red line was obtained by simply substituting very small values of the $P(C_{ik}=1)$ terms in the scenarios that correspond to element 20, rather than actually computing the Structural Risk again with a lower probability of Local Damage occurrence (Event Control) and/or with elements 20 and 2 reinforced (Specific Load Resistance).

The resulting Structural Risk of this modified structure is $P(C=1)=9.11\cdot10^{-6}$/yr.
This value is still higher than the target value $P(C=1)_{acc}=10^{-7}$/yr, but the choice of this element is more effective in reducing the Risk than any other element (i.e., by applying the Event Control and/or the Specific Load Resistance strategies to any other element, the reduction of the Structural Risk would be smaller than with element 20).

The same information would be useful with the Alternate Load Path strategy, to figure out which elements need to be improved in order to bridge over a critical area of the structure. In the present example, if the values of the $P(LD_{ik})$ terms relative to element 20 cannot be reduced, structural measures can be applied to decrease the corresponding Structural Vulnerability terms $P(C=1|LD_{ik})$.

It was observed (section 6.2.2.1, figure 6.23) that for lower values of the Central Safety Factor the estimates of the Structural Vulnerability terms $P(C|LD_{ik})$ obtained with standard and simplified FORM procedures are in the same order of magnitude, while for higher values of the CSF the difference becomes one or two orders of magnitude. This could lead to a considerable difference in the estimates of the Structural Risk $P(C=1)$ with the different FORM procedures.
To take into account this possibility, the Structural Risk $P(C=1)$ was calculated again, this time with the Structural Vulnerability terms $P(C|LD_{ik})$ divided by 100 in the cases where the Central Safety Factor is higher than the value 2.0 (and leaving the terms unvaried in the other cases). The resulting estimate of the Structural Risk is virtually equal to the first one; the difference between the two estimates results $7.30\cdot10^{-14}$/yr, i.e nine orders of magnitude smaller than the value of the Structural Risk itself $P(C=1)=3.95\cdot10^{-5}$/yr.
The very small difference is explained by noticing that higher values of the Central Safety Factor correspond to lower orders of magnitude of the probability of failure P_f; as a consequence, the more approximated terms of the summation (5.4) are much smaller than the other terms.

Completing the automatic analysis required a total of 337 model runs (the effective number of model runs is higher than the number of analyzed cases; this is because, as explained in section 6.2, in the static analyses a tentative maximum load is applied and, if Collapse condition is not reached, the computation is repeated with the maximum load doubled).
The analysis had to be restarted three times because Local Damage cases 64, 199 and 202 did not converge; for this cases a prudential value $P(C|LD_{ik})=1$ was adopted.
The total computation time resulted 3 hours and 42 minutes. On average, each model run required 40 seconds of computation time and each scenario required about 1 minute.

Section 5.3.1.2 describes a method to determine how many Local Damage scenarios need to be calculated to assess if a structure is acceptable or not. In the present example, calculating the LD1 scenarios alone would have been enough to ascertain that the structure is not acceptable, because their contribution to the Structural Risk sums up to

$$P(C=1)_{min} = \sum_{0}^{1}{}_{i} P(C=1|LD_i) P(LD_i) = 1.95 \cdot 10^{-5}$$

Calculating in automatic the LD0 scenario and the 30 LD1 scenarios would have required about 30 minutes.

6.3.2 Nine-story structure

A tentative testing of the methodology to calculate the Structural Risk was performed on the nine-story structure described in section 6.2.3.
The automatic algorithm was set to calculate the mean resistance of the structure in 135 scenarios, corresponding to all the LD1 scenarios in which one column is removed. The computation stopped at the 63rd scenario.

In the partial test a total of 122 model runs were performed, with a computation time of 27 hours and 39 minutes. On average each model run required 13.5 minutes of computation time and each scenario required about 26 minutes.
Based on this data, it can be estimated that:

- completing the calculation for the 135 LD1 scenarios in which one column is removed would require about 2.5 days;
- for all the 342 LD1 scenarios, the computation time would be about 6 days;
- for the 921 LD1 and LD2 scenarios in which columns are removed, the computation time would be about 16.5 days;
- for all the 1164 required LD1 and LD2 scenarios, the computation would require about 21 days.

6.3.3 Structural Risk calculation - Summary

The previous sections presented an example in which the first proposed methodology to calculate the Structural Risk (section 5.2.2) is tested on the three-story structure described in section 6.2.2, as well as an incomplete test on the nine-story structure described in section 6.2.3. The tests were aimed at assessing the feasibility and computational effort of the methodology, rather than studying the structure itself.

In section 6.3.1 the methodology is implemented on the three-story structure.
A total of 416 scenarios were considered, of Local Damage level ranging from 0 to 5, to which a semi-fictitious probabilistic characterization was given. The Structural Vulnerability terms $P(C|LD_{ik})$ were automatically calculated using the simplified FORM procedure and static non linear analysis. These parameters were used to estimate the Structural Risk of the structure by means of equation (5.4).
By comparing the single terms of the summation (5.4) it was highlighted the areas of the structure were the mitigation strategies (described in chapter 2) are most effective in order to reduce the Structural Risk.

It was observed that for high values of the Central Safety Factor the estimates of the Structural Vulnerability obtained with standard and simplified FORM procedures are in different orders of magnitude, but these terms give minimal contribution to the total Structural Risk because they correspond to the smaller terms of the summation.

The time required to perform the complete computation was in the order of hours.

A tentative testing of the methodology on the nine story structure is described in section 6.3.2.
A total of 135 scenarios were considered, corresponding to all the Local Damage level 1 scenarios in which one column is removed. The computation stopped at the 63rd scenario.
Based on the performed calculations, it was estimated that the time required to perform the complete computation would be in the order of days for the considered LD1 scenarios, and in the order of weeks if also the LD2 scenarios were considered.

6.4 Summary

This chapter presents some test applications of the first proposed methodology.
The used structural models are described in section 6.1
Examples in which the Structural Vulnerability is calculated on three different structures are presented in section 6.2.
Two examples of Structural Risk calculation are presented in section 6.3.

The following chapter summarizes the entire work and presents its conclusions.

Chapter 7 - Summary and conclusions

In this chapter, section 7.1 summarizes the entire work and section 7.2 presents its conclusions.

7.1 Summary

The present work is about Progressive Collapse of structures. It can be conceptually divided in two parts.
The first part, which comprises chapters 1, 2, 3 and 4, provides information to understand the problem this work is about and the proposed approach to cope with it.
The second part, composed of chapters 5, 6, 7 and 8, presents the proposed approach, some test applications of it and some ideas for further developments.

Chapter 1 introduces the concept of Progressive Collapse, lists the characteristics that are desirable in a structure in order to mitigate the phenomenon, and lists the causes of Collapses, subdivided in categories. Furthermore, it describes and analyzes some of the most important case studies of Progressive Collapse, as well as some cases in which an initial Damage did not evolve in a Collapse.
It is highlighted that the generally accepted definitions of Progressive Collapse include some ambiguities, and that the present work intends to overcome this problem by incorporating Progressive Collapse in a probabilistic Risk framework.

Chapter 2 lists and explains the strategies that have been devised for Progressive Collapse mitigation and lists some of the most significant regulations about this subject, explaining how they evolved.
It is highlighted that the level of attention given to Progressive Collapse has changed through the years, as it was boosted after the occurrence of the most significant cases but decreased in the following years.
Furthermore, it is pointed out that many research results that are still currently referenced are decades old and might not be fully valid. Likewise, some building codes requirements are still in force with little or no modifications, and their validity has been questioned.

Chapter 3 lists and analyzes several ideas that have been proposed for parameters to quantify the propensity to Progressive Collapse of structures, which should be useful to decide if mitigation methods need to be applied, and elaborates on the reasons why no building code has adopted any of these methodologies yet.
The chapter also elaborates on the terms “robust” and “Robustness”, which are widely used in the literature about Progressive Collapse but might lead to ambiguities because they can have different definitions and meanings.

Chapter 4 introduces the concept of Risk. It describes the probabilistic Risk management framework developed by the University of Braunschweig, whose concepts are used in this work, and points out some aspects of the adopted nomenclature.
Several facts that make it difficult to incorporate Progressive Collapse in a Risk framework are described and analyzed.

Chapter 5 presents two methodologies to quantify Progressive Collapse propensity of frame structures. The motivations to the development of these methodologies and the targets they aim to are described. The basic ideas of the methodologies are first presented, and then the details of how the methodologies can actually be implemented are described.

Chapter 6 presents some test applications of the first proposed methodology.
First the used structural models are described. Then examples in which the Structural Vulnerability is calculated on three different structures are presented, and then two examples of Structural Risk calculation are presented.
It is pointed out that the main target of the performed analyses is not the testing of the structures, but the testing of the algorithms used for the analyses, to study the the feasibility of the proposed approaches, to find out their problems, and to debug the implemented algorithms.

Chapter 7 summarizes the entire work and presents its conclusions.

Chapter 8 lists several aspects that need to be further considered to improve the proposed methodologies and elaborates on some of them. Other ideas that might be developed are also described.

7.2 Conclusions

The target of the present work is expressed in section 5.1 and here reported again:
attempting to understand if, and up to which extent, it is possible to devise a reliable and practical method to quantify the propensity to Progressive Collapse of a given structure, as well as to quantify an acceptable level of this propensity.

Every element in this statement has a precise reason to be. Let's analyze the statement:

- the core concept is that two things are needed: the quantification of the propensity to Progressive Collapse and the quantification of an acceptable level of this propensity.
- The target of the present work is attempting to understand if it is possible to devise a methodology to obtain these two things.
- Two constraints are added: the methodology should be reliable and practical. In other words, it should provide information that really represents the studied system, and it should be usable in practice. These constraints derive from the consideration that some quantification methodologies found in literature don't have these qualities, which makes them virtually pointless or unusable.
- The target of the work can be expressed with the question "is it possible to devise such methodology?". The answer to this question cannot simply be a "yes" or a "no", because every methodology can provide different levels of information quality, different levels of reliability, and different levels of practicability. This is why the words "*and up to which extent*" are included in the statement. Studying "*up to which extent*" the target can be reached implies understanding the limitations and the problems of the chosen approach, which consequently will influence the way the methodology is constructed.

Now the obvious question arises: was the proposed target achieved?
Not yet. The first steps towards the achievement of the target were performed. The developed methodologies are to be considered as prototypes that need further improvement to actually be usable.

Another question that might arise is about the effective validity of the proposed approach. Is it really valid?
The proposed approach consists in incorporating Progressive Collapse in a probabilistic Risk framework; this type of approach proved to be effective in reducing Losses with other types of

Hazards. No particular evidence was found to prove that the target of the present work is impossible to reach through this approach.
There are two aspects that might lower the validity of the approach.
The first is the availability of data to formulate a probabilistic characterization of the Local Damage scenarios. If this characterization is not obtainable, then a conventional (or partially conventional) one could be used, at the cost of a reduction of how well the results represent reality.
The second aspect is that deciding an acceptable level of the Structural Risk depends on the acceptable Total Risk, i.e. on the expected Losses. The relation between the two types of Risk generally depends on the specific context and is not studied in the present work. The quantification of an acceptable level of Risk is important because without it the methodology cannot be a decision tool: the question "*is this structure sufficiently safe?*" cannot be answered if there is not an absolute reference to compare the calculated parameters to.

One more question: if it will be proved that the methodology is effective, will it be necessary to apply it to every structure?
The answer is: no. First of all, it is reasonable that estimating the propensity to Progressive Collapse would be useful only for those structures whose Collapse can produce consistent Losses, such as strategic and/or very big buildings.
Secondly, if the ultimate target of a Progressive Collapse study is reducing the Risks at an acceptable level, then the best imaginable scenario is to prove that the indirect method is effective to achieve that target. In other words, the best scenario is that, by prescribing some structural measures, there is certainty that the structure is sufficiently safe, without the need to perform any calculation. This might be possible at least for structures of a given regularity, size and importance, similarly to what is commonly done in seismic designing.

Chapter 8 - Further aspects to consider

This chapter presents several aspects that need to be further considered in order to improve the proposed methodologies, as well as other ideas that might be developed.

8.1 Improvements and testing needs

As stated, the proposed methodologies are to be considered as a first step towards the achievement of the proposed target. This section contains a list of aspects that need to be further studied; the following sub-sections elaborate more in detail some of these aspects

Dynamic effects. If occurrence of a Local Damage is sudden, then the dynamic effects in the structural behavior should be non negligible. Section 8.1.1 discusses on how they can be taken into account.

Discretization. In the implemented models, all beams are modeled with 10 beam type elements and all columns are modeled with 5 beam type elements, in order to capture the effects of geometrical nonlinearities. This discretization was adopted because it is common practice at the National Institute of Standards and Technology, where the original models were developed. Depending on the type of finite elements used, a denser discretization might result in more accuracy of the results, while a coarser discretization should make the models smaller and faster. Further testing is needed to find out an optimal discretization level (see also section 8.1.2).

The "immaculate removal" approach. In the performed tests, Damage was always modeled by removing structural elements from the structure and retaining integrity of the adjacent nodes, following the so called "immaculate removal" approach (section 2.2.2.3, figure 2.7). This approach was adopted for its simplicity, but might not represent real events correctly. Section 8.1.3 further discusses this issue.

Progression of the Damage. In this work two methodologies to reach the proposed target are presented (sections 5.2.2 and 5.2.3, respectively). The difference between the two is the type of information that is obtained: in the first one the incipit of Collapse is considered, in the second one the final extension of the Collapse.
Only the first methodology was actually implemented and tested. The second was not, for two main reasons. The first one is that a model that simulates the progression of the Collapse was not available. The second is that the complexity and the number of unknowns are much bigger than in the first methodology; it was preferred to study the first one to get experience and knowledge from it. Section 8.1.4 deals with the modeling of Damage progression.

Interaction with non structural elements, such as wall panels and floor slabs, can have a considerable influence in the behavior of a structure (see for example figure 2.2). Research is needed to assess how different the results can be by neglecting this interaction, and to find out simplified methods to include it in the structural models. One idea to follow could be to include struts in the structural models to simulate the effect of the panels, as it is already done in seismic engineering (figure 8.1).

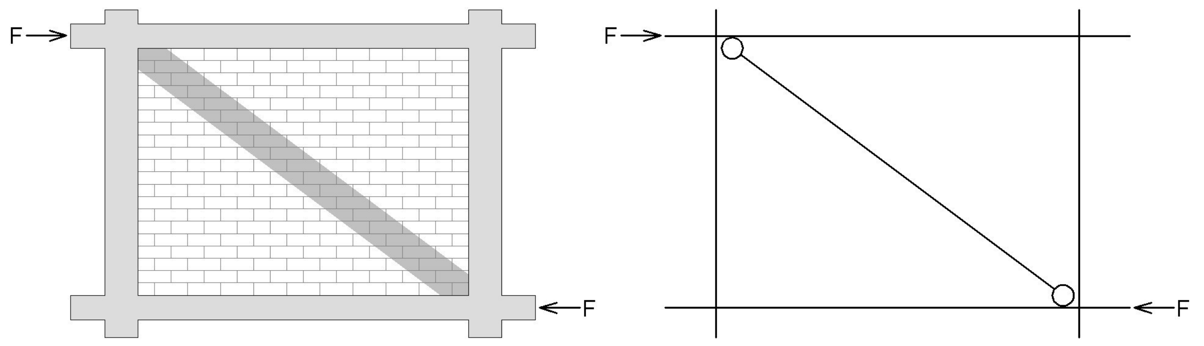


Figure 8.1 : Representation of how the effect of a wall panel in a frame (left) can be modeled with a strut element (right).

Instability. Structural Damage can spread as a consequence of a loss of stability, as in the case of Building 7 of the World Trade Center (section 1.4.4.2; figure 1.23). The algorithms implemented for the presented tests consider the detachment of a structural element as Collapse condition, but do not take into account column buckling.

The characterization of the random variables must be further studied. In the presented examples, the characteristic values of the loads and of the material are taken from the original design, while their probabilistic characterization (type of distribution and relationship between the relative parameters), as well as the characterization of the model uncertainty term ξ, are taken from literature. There is no guarantee that these are the best choices to represent the analyzed structure.

Number of random variables to consider. In the performed tests, four or five random variables were considered. In order to obtain reliable enough results, the number of variables that really need to be considered could be much higher.
In particular, a single random variable has been assumed for the live load L and one for the dead load D, which have been considered uniformly distributed on all the floors of the analyzed structure; it seems reasonable that at least a different variable of each type is needed for each floor. Val's methods [50] can be implemented to take into account different uniformly distributed loads on each floor, but they cannot take into account different load patterns, like for example “chessboard” type loads.
Also, in the performed tests only one random variable that characterizes the materials is used (the yield stress of steel f_y). Further research is needed to find out if other properties need to be considered and how the variation of the material properties in different part of the structure can affect the results.

Other reliability methods. The implemented methodology uses the First Order Reliability Method, which is more approximated but faster than other methods. Other less approximated methods could be incorporated in the methodology.

More tests are needed in order to further validate the procedures, as well as to assess the influence on the results of details such as the adopted load step sizes, the stop criteria parameters, the types of finite elements, and so on.

More experimental results are needed in oder to simulate correctly the behavior of the structural elements.

Fire can have an important role in Progressive Collapses. For example, all the three World Trade center Collapses (section 1.4.4) would not have happened in absence of fire. Taking into account fire would require the introduction of the variable time in the problem, i.e. analyzing how the characteristics of the structures vary in time.

Other construction technologies. The presented methodologies are restricted to frame structures and were tested on steel frame models. They need to be improved in order to be applicable, for example, to masonry structures.

8.1.1 Dynamic effects

In the presented examples, the Structural Vulnerability terms $P(C|LD_{ik})$ are calculated using static nonlinear analysis. In reality the occurrence of a Local Damage is likely to be sudden; as a consequence, the dynamic effects should be non negligible and the actual value of the $P(C|LD_{ik})$ terms should be higher than the ones calculated. This section briefly elaborates on how they can be considered.

One way to take dynamic effects into account is to apply amplification coefficients in static analyses. This approach is commonly used in Alternate Load Path verifications. Typically, the static analysis is performed with the loads in the damaged area multiplied by a coefficient. This coefficient is often taken equal to 2, because in a vertical single-degree of freedom (SDOF) linear elastic oscillator in free vibration the maximum reaction is twice as the static reaction (figure 8.2). Examples of this are the formulas (2.7) and (2.9), which are from the regulations DoD 2005 [12] and GSA 2003 [15], respectively.
The value 2 might be excessively approximated. Studies have recently being carried out to find out more realistic values (McKay et al. [30]), which have been incorporated in the 2009 version of the DoD regulations [13] (formula (2.8)).

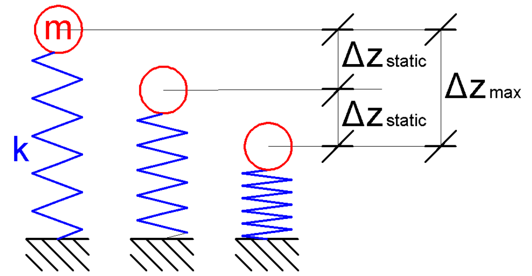


Figure 8.2 : Schematic representation of a vertical linear elastic SDOF oscillator. The maximum vertical displacement is twice as the static displacement. As a consequence, the maximum reaction is twice as the static reaction.

The issue can also be addressed by simply performing a dynamic nonlinear ("time history") analysis, instead of static. The analysis consists in modeling the loaded undamaged structure, then suddenly remove the "failed" elements, and then check if the Collapse condition is reached in the following moments.

Compared to static analysis, implementing the presented Structural Vulnerability calculation procedures with dynamic analysis implies one big practical difference: the value of the resistance R (and consequently of the performance function g) for a given set of random values cannot be calculated with a single model run. Instead, only the sign of g can be calculated (negative if Collapse condition is reached, positive if it is not).
As a consequence, in the standard FORM procedure the Newton method cannot be used to reach the critical condition $G(r,\varphi)=0$ by moving along the r polar coordinate (see section 5.3.1.1.3 and figure 5.5), and a slower method like bisection must be used instead.
For the same reason, applying dynamic analysis to the simplified FORM procedure would require more runs of the model in order to estimate the mean value of the resistance μR, while static analysis only requires one for each Local Damage scenario.

8.1.2 Accuracy

Predicting the behavior of a structure requires modeling it, and models can be implemented using different methods and different levels of detail.
The more a model is detailed, the more accurate the results are likely to be. The drawback of very detailed models is the big effort required to implement and run them, which can make them virtually impossible to use (some models used in the context of Progressive Collapse literally required weeks or months of computation time for just a single run; see for example those described in sections 8.1.4.3 and 1.4.4.2).
Conversely, simpler models require less effort, but they might neglect important details and thus provide inaccurate results.

To achieve the proposed targets by means of the presented procedures we need models that are accurate enough, but do not require excessive computational effort. What “accurate enough” means is subjective; the following are some considerations about this.

Accuracy is commonly defined as the degree of closeness of a measured or calculated quantity to its actual (true) value. Let's say that it is possible to witness a physical phenomenon and measure the actual value of an interesting parameter IP_A. The “degree of closeness” of the calculated parameter IP_C can then be expressed with the relative error

$$E_{AC}=|(IP_C-IP_A)/IP_A| \tag{8.1}$$

Saying that a model is “accurate enough” means that the relative error is always within an acceptability range, for all possible scenarios that need to be considered. The subjectiveness consists in deciding how wide this acceptability range should be.
Also, two different models of the same physical phenomenon can be compared by calculating the relative difference:

$$E_{12}=|(IP_{C2}-IP_{C1})/IP_{C1}| \tag{8.2}$$

If model 1 is more detailed than model 2, then we can assume that the former will produce more accurate (closer to reality) results then the latter. Then the relative error of model 2 is[1]

$$E_{A2}=|(IP_{C2}-IP_A)/IP_A|=|(1\pm E_{12})\cdot(1\pm E_{A1})-1| \tag{8.3}$$

In the light of these considerations it is possible to sketch a procedure to asses if the accuracy of a model is acceptable.

The first thing to do is to choose one parameter (or more parameters) of the physical phenomenon that is (are) interesting for the proposed goal.
In the present work, the modeling is required to estimate the Structural Vulnerability terms $P(C|LD_{ik})$ (first proposed procedure; section 5.2.2) and $P(FD=f|LD_{ik})$ (second procedure; section 5.2.3). A choice of the interesting parameter to consider for the first procedure can be the resistance R (i.e. the load that, applied to the structure, causes spread of the Damage); for the second procedure, it can be the final extension of the Damage FD.

Then an acceptability range AR (i.e. the maximum acceptable value of the relative error E_{AC}) must be chosen. For example, it can be AR=0.10=10%.

If it is possible to witness the physical phenomenon and obtain a measure IP_A of the interesting

1 The signs are: $(1+E_{A1})$ if $IP_{C1}>IP_A$; $(1-E_{A1})$ if $IP_{C1}<IP_A$; $(1+E_{12})$ if $IP_{C2}>IP_{C1}$; $(1-E_{12})$ if $IP_{C2}<IP_{C1}$.

parameter, then from (8.1) follows that a model can be deemed accurate enough if

$$E_{AC} \leq AR \tag{8.4}$$

for all possible scenarios.
In general it is not possible to witness the physical phenomenon, or it is possible just for a few scenarios. In these cases the accuracy check can be performed by comparing the model with a very detailed one. If model 1 is so detailed that the relative error E_{A1} can be considered negligible compared to the relative error E_{12}, then eq. (8.3) becomes

$$E_{A2} \approx E_{12} \tag{8.5}$$

and the model can be deemed accurate if

$$E_{12} \leq AR \tag{8.6}$$

for all possible scenarios.

In order to develop models that are accurate enough, but do not require excessive computational effort, one way (probably the only way) to follow is to start from experimental data and very detailed models and simplify as much as possible, keeping track of the errors introduced in every simplification step.
In the present context, the adjective "detailed" refers to the capability of the used elements to represent the analyzed phenomena, to the level of discretization of the modeled system and to the number of considered parameters and properties. For example, a single structural beam can be modeled with many 3D elements, or with several 1D elements, or with just one 1D element; with some softwares these different levels of discretization can lead to noticeably different results (as in the example of figure 8.3).

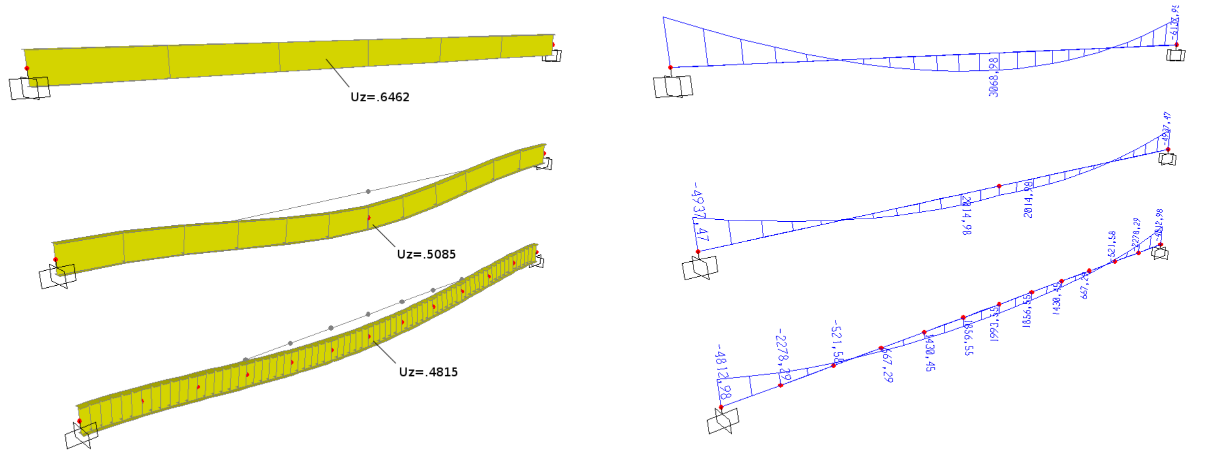


*Figure 8.3 : The same beam, with fixed ends and subjected to the same uniform load, modeled with the software SAP2000 using one (first row), two (second row) and twelve (third row) 1D elements. The material model is linear elastic and the analysis is set to include geometrical nonlinearities. (Left) The difference in the calculated maximum deflection between the first and the third case is 34% and between the second and third case is 6% (in the first case the graphical interface is not able to draw the deformed shape). (Right) The difference in the maximum (positive) bending moment is respectively 54% and 1%; the difference in the minimum (negative) bending moment is respectively 28% and 3%. It is worth to highlight that the DoD 2005 regulation [12], in its section C-5 presents an example of Alternate Path analysis using this software, stating that "*if large displacements are used, it is very important that every member that forms a plastic hinge is subdivided into at least 20 smaller members. This is the only way SAP can determine the catenary effects*".*

As for the choice of the acceptability range AR, it does not necessary need to be "very small". The important is to be aware of the level of approximation of the calculated results (or, in other words, of the maximum "distance" that can be between the calculated value of the interesting parameter and its actual value).

It must also be noted that it does not matter if other, "non interesting" parameters of the physical phenomenon are not represented accurately by the model (as long as we are aware of this, of course).
A similar "pragmatic" type of approach is widely used, for example, in seismic engineering, where linear analyses can be used to assess if a structure is safe enough. When using this type of analyses, the designer must always be aware that the used models do not replicate the actual phenomenon; the maximum displacements calculated directly from the models, for example, are usually underestimated.

Since in the presented methodologies the ultimate target is the calculation of the Structural Risk $P(C=1)$ and $P(FD=f)\cdot f$, the choice of the acceptability range AR should depend on the acceptable approximation of these quantities.
For example, in the first methodology the model is used to calculate the critical load IP_C of a given scenario, which in turn is used to calculate the Structural Vulnerability $P(C=1|LD_{ik})_C$.
From (8.1) follows that the actual value of the critical load is $IP_A=IP_C/(E_{AC}+1)$.
As a consequence:
- the actual value of the Structural Vulnerability $P(C=1|LD_{ik})_A$ is in a neighborhood of the calculated value $P(C=1|LD_{ik})_C$,
- and the actual value $P(C=1)_A$ is in a neighborhood of the calculated value $P(C=1)_C$.

The maximum acceptable radius of this last neighborhood should be the criterion to choose the acceptability range AR of the interesting parameter.

In the literature about Progressive Collapse (scientific papers and building regulations) there are models with every level of detail. The Writer is currently not aware of any study about the influence of detail level on results.

8.1.3 About the quantification of the Damage Level

In the presented methodologies the Damage levels are always expressed as "*the number of "failed" structural elements, i.e. as the number of elements that are not able to fulfill any of their functions anymore*". Furthermore, in the first procedure for Structural Vulnerability calculation it is assumed that Collapse condition occurs if at least one connection in the structure is lost, resulting in physical separation (section 5.3.1.1). These assumptions deserve some observations.

In the presented examples the Local Damage scenarios are modeled by removing some structural elements while retaining the integrity of the adjacent nodes. This approach, commonly referred as "immaculate removal", is adopted mainly for its simplicity (section B-4.1 of DoD 2005 [12], here reported in section 2.2.2.3; see also figure 2.7).
This assumption might not represent real events correctly. It corresponds to case (a) of figure 8.4. Cases such as (b), (c) and (d) can also happen; they would generally result in lower critical loads and, ultimately, in higher values of the calculated Structural Risk.

Further research is needed to assess the influence of the immaculate removal assumption in the Risk analysis. An extensive survey of the type of Damage caused by different Hazards might highlight if some types of nodal Damage are more frequent. It is likely that structural details have a strong influence on this; for example, if the structure is reinforced concrete with little confinement and insufficient continuity of the longitudinal bars, then case (d) is much more likely to happen then the other ones.

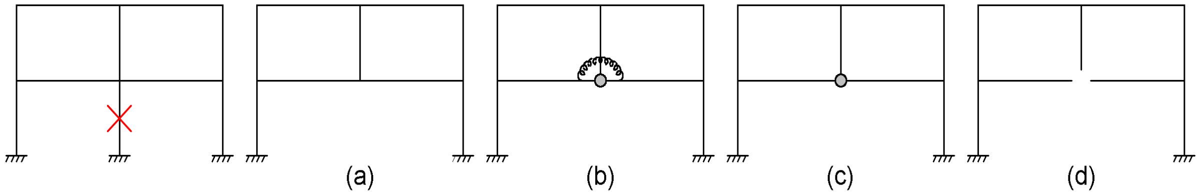


Figure 8.4 : Four different ways to model the removal of an element from a structure.

Another issue is how you quantify the final extension of the Damage. In the second proposed procedure, it is assumed assumed that "*the final extension of the Damage is expressed as the number of failed structural elements*" (section 5.2.3).
It must be highlighted that, according to the adopted definition of "failed" element, the simple detachment of a connection does not necessarily imply that the adjacent elements are failed. For example, in case (a) of figure 8.5 the element would not be considered failed if the remaining connection is able to bear the loads.

Furthermore, the number of failed elements is not the best parameter for the estimation of Losses; expressing the extension of the Damage as the failed area would be better. In fact, if we compare a structure with many small structural elements and one with few big elements, the same number of failed elements will correspond to a different failed area and, ultimately, to different Losses and a different Total Risk.
Moreover, two elements can have different importance, even if they have similar dimensions; e.g. the examples of figure 8.6 depict the failure of beam elements of similar length, but the example on the right does not imply Collapse of floor area.

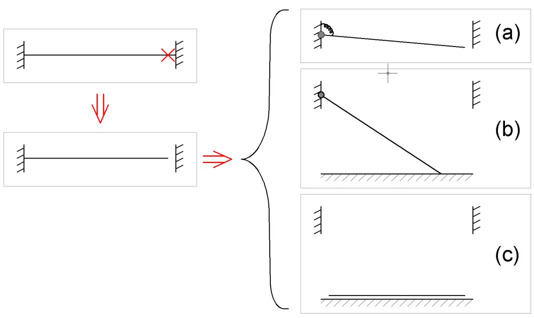


Figure 8.5 : Three possible consequence of the detachment of a beam connection.

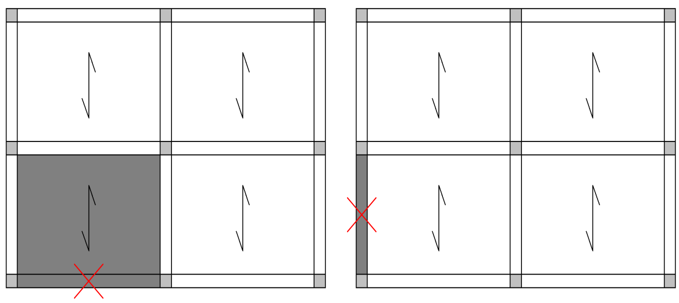


Figure 8.6 : Two examples of failed beams that lead to different consequences.

8.1.4 About the modeling of Damage progression

Applying the procedure described in section 5.2.3 requires modeling the progression of the Collapse. Some building regulations (e.g. DoD 2005 [12] and GSA 2003 [15]) include procedures for Alternate Load Path verification, in which progression of the Damage is considered. The effective reliability of these procedures is unknown, since they are deterministic and the modeling of Damage progression is very simplified. (It must also be noticed that the 2009 version of the DoD regulation [13] does not admit Damage progression anymore).

Nowadays the Finite Elements Method (FEM) is probably the most widely used numerical technique for structure modeling. It has been studied for decades, and most current FEM softwares can handle advanced features such as material and geometrical nonlinearities and structural instability. Some FEM codes allow to remove elements from the model, even automatically during a computation; this feature is sometimes used to simulate the "failure" of structural elements in Progressive Collapse analyses (e.g. in [31]).

The three following sub-sections briefly describe some other approaches and methods that might be usefully applied to the considered problems.

8.1.4.1 Applied Elements Method (AEM)

The Applied Elements Method (AEM) has been invented by H. Tagel-Din in 1995 and it is described in many of his works (e.g. in [47]).
With the AEM, a structure is modeled as an assembly of small 3D elements, connected by springs located at contact points distributed around the element edges. Stresses and deformations are properties of the springs, rather than of the elements.
The Authors claim that the Applied Elements Method has more capabilities than the Finite Elements Method, which are especially useful in the field of Progressive Collapse (figure 8.7, right). In particular, it is said to be capable of modeling separation (even partial) and collision of the elements.

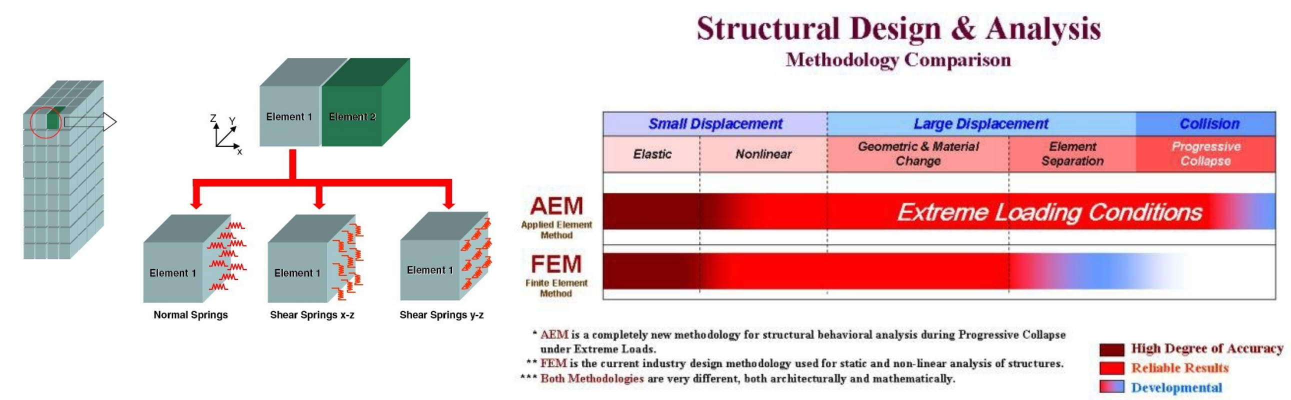


Figure 8.7 : (Left) Schematic representation of Applied Elements connectivity. (Right) Comparison of Applied Elements Method and Finite Elements Method capabilities, according to the advertising of an AEM software company. (Source: www.appliedelementmethod.com).

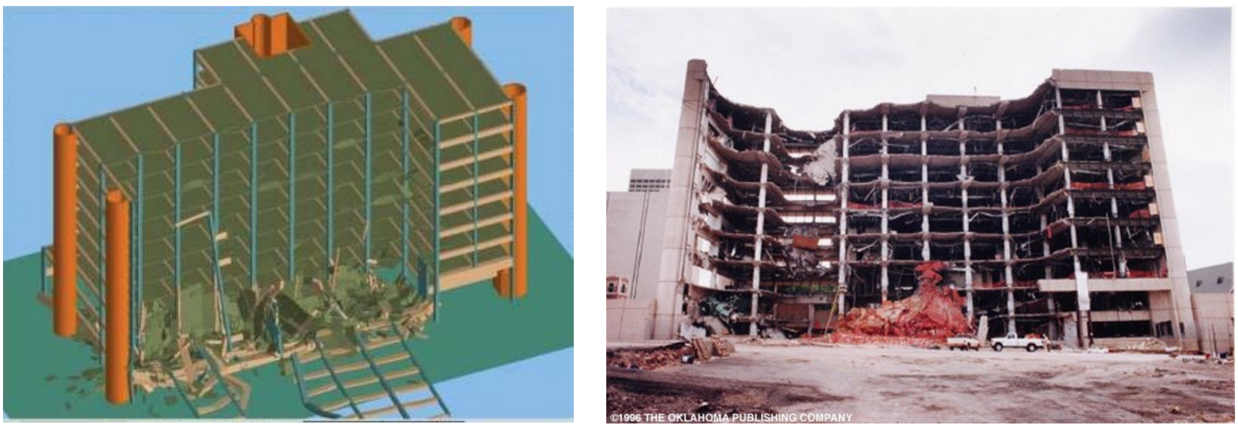


Figure 8.8 : The Alfred P. Murrah federal building of Oklahoma City, USA. (Left) Screenshot of a AEM model. (Right) The building after its Progressive Collapse. (Sources: www.appliedelementmethod.com; Oklahoma Publishing Company).

Several studies have already been carried out with the AEM. For example in [47] it is used to model the A. Murrah building Collapse (section 1.4.3). A rough idea of the required computation times of the method is given by the following data: using 10,000 3D elements and a time step of 0.0001 seconds, the analysis took around 4 days on a 3.2 GHz Pentium IV to reproduce about 6 seconds of Collapse.

8.1.4.2 Fuzzy logic and neural networks

Möller et al. [32] present a method to estimate the probability that a Collapse happens according to a given modality. The method uses the general uncertainty model fuzzy randomness and the structural response is approximated with a neural network following the response surface methodology.
An example is presented in [32]. A fictitious, three-story reinforced concrete building is modeled with the Finite Elements software LS-DYNA, using 2330 3D elements. Damage is modeled by removing from the computation the elements that reach a specific plastic strain; the Authors admit that "*...this element 'erosion' algorithm does not conserve mass, introduces element size dependent results and for compressed parts, does not allow to describe correct geometry. Nevertheless, for a large part of the collapse events mainly in the first important phases the negative effects of this erosion model are rather small*". Contact between elements is considered.
Each run of the FE model required approximately 20 min of CPU-time on one processor of an Itanium2 cluster; since a dataset of 500 computational results was used to train the neural network, the theoretical[2] total computation time was approximately 7 days.

The result of the computation is a "fuzzy probability", i.e. an interval inside the range between 0 and 1. In the presented example the probability that the Collapse happens according to the chosen modality "*...varies between 0.781 and 0.992*".

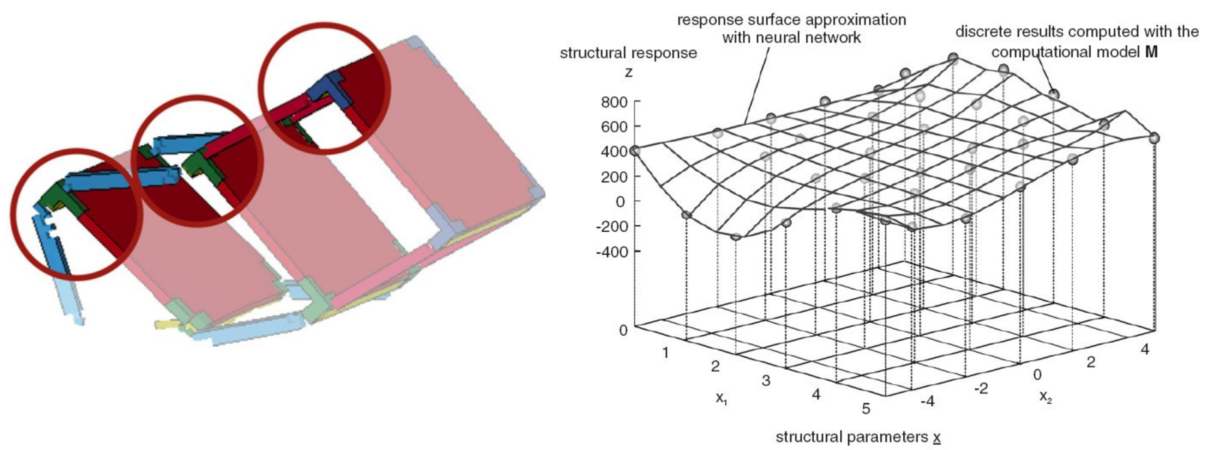


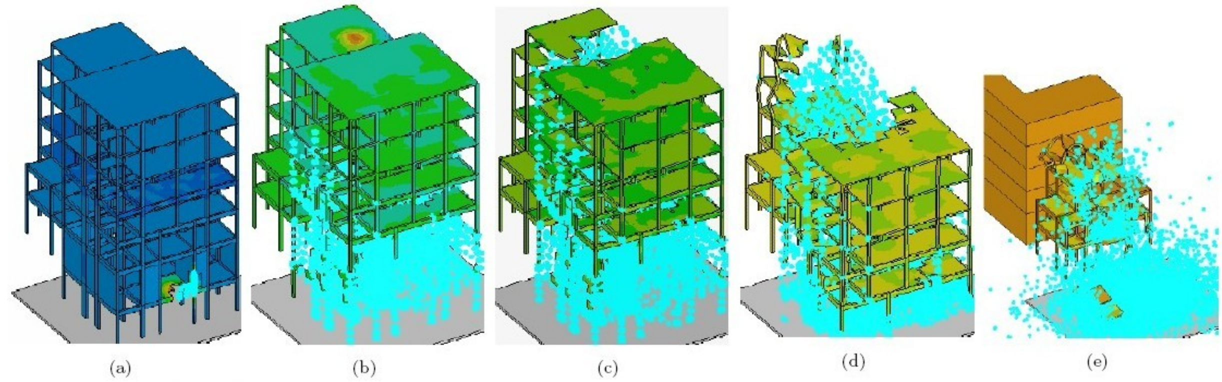
Figure 8.9 : Study by Möller et al. (Left) Finite Element representation of the collapsing structure. (Right) Approximation of the structural response with neural networks. (Source: [32]).

8.1.4.3 Hydrocodes

Luccioni et al. [26] use a hydrocode to reproduce the actual Collapse of a building that suffered a terroristic bombing (figure 8.10). A hydrocode is "*...a computational tool for modeling the behavior of continuous media. In its purest sense, a hydrocode is a computer code for modeling fluid flow at all speeds. It can, however, be adapted to treat material strength and a range of rheological models*

2 Itanium2 are multi-core CPUs.

for material behaviour" [10].



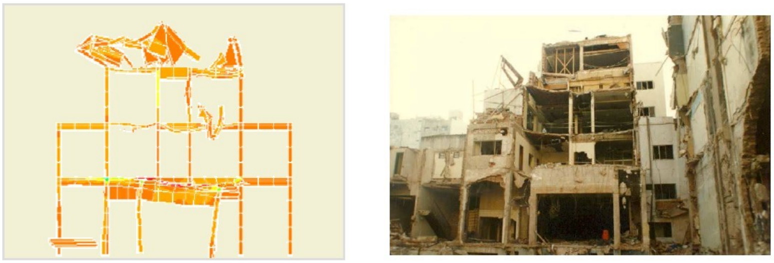


Figure 8.10 : The 1994 bombing of the AMIA building in Buenos Aires, Argentina. (Top) Screenshots of the hydrocode model of the Collapse by Luccioni et al. (Right) Comparison between the actual and the modeled final Damage. (Source: [27]).

In [26] columns, beams and slabs are modeled with 3D solid elements; walls are modeled with shell elements. The air in the structure is modeled, too. The model simulated the detonation and its direct Damage, as well as the subsequent Collapse. Running the model required approximately 310 hours (almost 13 days) on a machine with a Pentium IV Processor and 500 Mhz RIMM Memory. The paper does not mention the total number of elements used, the total time interval modeled and the time step of the computation.

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"Mentvla conatur Pipleium scandere montem: Musae furcillis praecipitem eiciunt"